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Persistence of the Sovereign Debt Components and Debt Sustainability:

Some Evidence for the US and Europe

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PERSISTENCE OF THE SOVEREIGN DEBT COMPONENTS AND DEBT SUSTAINABILITY: SOME EVIDENCE FOR THE US AND EUROPE

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Abstract

This paper analyses the persistence and mean reversion properties of sovereign debt and its components by applying fractional integration methods to long runs of annual data starting in 1831 for the UK and the US, in 1862 for Italy and in 1881 for France and Germany, and ending in all cases in 2022. The empirical results provide evidence of a high degree of persistence in all debt/GDP series, which appear to be I(1). However, cross-country differences emerge when analysing the properties of the individual components of debt. Specifically, in countries with a relatively large debt/GDP ratio, such as France and Italy, the primary balance is more persistent ($0.5<d<1$) than the snowball term in the budget constraint equation, while in those with lower debt, such as Germany and the US, it is stationary but bond yields are persistent ($0.5<d<1$). In all cases, the other financial transactions series is stationary and peaking in the war periods, while GDP growth exhibits low persistence and mean reversion ($0<d<0.5$). We also assess debt sustainability by analysing the stochastic properties of the differential between interest payments and primary deficits. The empirical findings ($0.4<d< 0.6$) support sustainability in all countries (both over the full sample and the most recent period from 1950), although mean reversion appears to be rather slow and a sizeable GDP growth might be needed to keep the debt/GDP ratio on a sustainable path.

Keywords: fractional integration; mean reversion; persistence; debt sustainability

JEL Classification: E52; C32

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1. Introduction

Analysing the evolution of public debt is of paramount importance to establish whether or not it is on a sustainable path. Prior to the pandemic, the global debt-to-GDP ratio had been increasing for decades, with global public debt tripling since the mid-1970s to reach 92% of GDP by the end of 2022 (IMF, 2023a), and peaking at $92 trillion in 2022 (UNCTAD 2023); in advanced economies, the public debt to GDP ratio reached 112% in 2023 (IMF 2023b).

The inflationary pressures of recent years (resulting in particular from the sharp increase in energy prices following the Russian invasion of Ukraine) have led to higher interest rates and cost of servicing the debt, adding to fiscal pressures and posing risks to financial stability (in addition to affecting growth and employment negatively). It is therefore essential to adopt fiscal measures to reduce global debt levels to more sustainable levels (IMF 2024a). For a long time, debt dynamics had not been a major concern, as interest rates were below the growth rate of the economy and therefore expanding fiscal deficits and debt stocks did not threaten solvency (Adrian et al., 2024). However, in the recent period of tight monetary policy and low growth debt sustainability has again come to the forefront.

In the US public debt is currently equivalent to 98% of GDP and it is estimated that the US government has 20 years to take corrective action, otherwise no amount of tax increases or spending cuts will be able to prevent debt monetisation from producing significant inflation (PWBM, 2023). From 1950 to 1970 public debt in the US was relatively low thanks to high economic growth and a restrictive fiscal policy; however, in the eighties the increase in military spending drove it upwards, and debt did not stop growing until the nineties, when budget surpluses were achieved thanks to a combination of more disciplined fiscal policy and higher economic growth. Since then, US public debt as a percentage of GDP has started growing again, driven by large budget deficits, and projections indicate that it will continue to grow over the next 30 years, reaching its highest level in 2029 (CBO, 2024).

As for Europe, when the Economic and Monetary Union (EMU) was launched in 1992 with the signing of Maastricht Treaty, this imposed a public debt ceiling of 60% of GDP to ensure that all member states would converge towards a sustainable debt level. However, the 2007-8 global financial crisis led to a sharp increase of debt in those countries (84% in 2023 for EU members and 89% for the Eurozone (IMF, 2023c) either through expansionary fiscal policy or unconventional monetary policy measures such as the Quantitative Easing (QE) adopted by most central banks in response to the crisis.

One important issue in this context is the degree of persistence of sovereign debt, which might shed light on whether or not corrective policies are required to ensure sustainability. In a previous study, Martin-Valmayor et al. (2024) found that this is linked to the persistence of the primary deficit. The current paper aims to extend their analysis by examining the possible role of a wider set of variables affecting the evolution of debt (though not of the primary balance – ECB, 2011) also including bond yields, the nominal GDP growth rate and other financial transactions. More specifically, long spans of data from the 19th century to the present are analysed for the US and the four main European economies, namely Germany, France, Italy, UK. It is noteworthy that there are some institutional differences between these countries which might have implications for the evolution of debt. Specifically, in all of them the government is responsible for fiscal policy; however, whilst the US and the UK are characterised by an independent monetary policy, the other countries in our sample belong to the eurozone and therefore in their case monetary policy decisions are made by the European Central Bank (ECB).

To estimate persistence we use fractional integration methods which are more general than standard models based on a dichotomy between stationary and non-stationary series since they allow the integration parameter to take any real value, including fractional ones, as opposed to integers only. As a result, a much wider range of dynamic processes can be considered and valuable information can be obtained on the persistence and mean reversion properties of the series.

We also address directly the issue of sustainability by following the approach of Trehan and Walsh (1991), namely by examining the stochastic properties of the differential between interest payments and primary deficits. For this purpose, we use again a fractional integration framework, which is more general than their approach based on unit root testing, and also extend their analysis by considering more countries and a much longer data span. In addition, we assess persistence using the budget constraint, which includes the GDP growth rate, a variable which might play a crucial role in ensuring sustainability.

To sum up, the contribution of this paper is twofold. First, it provides more extensive evidence on the roots of debt persistence and also on whether or not public finances are on a sustainable path using long runs of data for various countries. Second, it adopts an econometric approach that is more flexible than the unit root testing generally carried out in previous studies on this topic. The rest of the paper is structured as follows: Section 2 provides a brief review of the relevant literature; Section 3 outlines the fractional integration approach used for the analysis; Section 4 describes the data and discusses the empirical results; Section 5 offers some concluding remarks.

2. Literature Review

The literature aiming to assess the persistent behaviour of sovereign debt is very extensive. Several studies have used unit root testing (Antonini et al., 2013; Camarero et al. 2015; Brady and Magazzino, 2017; Campos and Cysne, 2022 among others) finding evidence in all cases of unit roots, which implies that shocks have long-run effects. These papers used linear tests such as the Dickey and Fuller (ADF, 1979) or Phillips and Perron (PP, 1987) ones, or non-linear and more efficient ones such as Kwiatkowski et al. (KPSS, 1992) or Elliot et al. (ERS, 1996). Other studies which have tested for debt sustainability using unit root tests to establish whether or not mean reversion occurs include Trehan and Walsh (1991), Caporale (1992), Chen et al. (2018), Feve and Henin (2000), and Uctum and Wickens (2000). To assess solvency, Caporale (1995) used a different method initially developed to detect speculative bubbles in financial markets and tested whether the government's budget is intertemporally balanced in a number of European countries; he reported that in some of them (Italy, Germany, Denmark and Greece) the government was not intertemporally solvent (see also Caporale, 1997, for another study focusing on the budget constraint to test for solvency). More recent papers have adopted a cointegration approach with or without structural breaks (Bajo-Rubio et al., 2010, Baharumshah et al 2017), in some cases testing for cointegration between public expenditure and revenue (Escario et al., 2012; Camarero et al., 2013).

However, it has become apparent that unit root tests do not provide reliable evidence. For instance, Diebold and Rudebush (1991) and Hassler and Wolters (1994) examined the properties of the Dickey-Fuller tests under fractionally integrated alternatives and showed that they have low power of the tests under this type of alternatives. Similarly, Lee and Schmidt (1996) examined the KPSS tests and found evidence of unbiasedness only against stationary long memory alternatives or $0 < d < 0.5$.

The issue is that imposing a dichotomy between I(0) and I(1) behaviour is very restrictive as many series exhibit long memory and are non-stationary but still mean-reverting, which occurs if the differencing parameter is in the range [0.5, 1). In such cases, fractional integration is the most appropriate modelling framework as standard unit root tests would lead to the incorrect conclusion that such series exhibit unit roots.

Only a few studies have followed this approach to examine the persistence of sovereign debt. One example is the paper by Cuestas et al. (2014), who used this method to analyse sovereign debt dynamics in the original twelve euro area members over the period 2000-2013. They found evidence of high persistence and no mean reversion in France, Germany and Italy, and in the case of the latter country they also detected a structural break in 2008. In another study using fractional integration techniques Caporale et al. (2021) analysed the private debt to GDP ratio in 43 OECD countries (including all G20 and BRICS) for the period 1951-2020. This series was found to be highly persistent in all countries except Argentina, with orders of integration around or above 1 (6 with unit roots or $d = 1$, and 37 with I(d) structures with $d > 1$). More recently, Martin-Valmayor et al. (2024) analysed the long-term behaviour of the debt/GDP and debt-per-capita ratio in the US and the five largest European economies and provided evidence of a high degree of persistence in both series, which were found not to be mean-reverting. The present study belongs to the same area of the literature, but takes the analysis further by estimating a fractional integration model for more variables and using longer data spans, as well as analysing in greater depth the roots of debt persistence and testing directly for its sustainability, as detailed in the following sections.

3. Empirical Framework

The empirical analysis carried out in this paper is based on a fractional integration framework which allows the differencing parameter d to take any real value, including fractional ones. Within this context one can define a covariance or second-order stationary process $\{x_t, t = 0, \pm 1, \ldots\}$ with mean μ as integrated of order 0 and denoted as I(0), if the infinite sum of the autocovariances, calculated as $\gamma(u) = E[(x(t) - \mu)(x(t+u) - \mu)]$, is finite, that is:

$$\sum_{j=-\infty}^{\infty} |\gamma(u)| < \infty. \qquad (1)$$

These types of processes are said to exhibit short memory and include not only the white noise but also the stationary and invertible AutoRegressive Moving Average (ARMA) model, which is frequently employed for stationary series. By contrast, a process is said to exhibit long memory (so-named because of the relevance of observations in the distant past) if the infinite sum of its autocovariances is infinite:

$$\sum_{u=-\infty}^{\infty} \gamma_u = \infty. \qquad (2)$$

There exist many models that satisfy the above condition, including those based on fractional integration or I(d) with d > 0. In such a case x_t is said to be integrated of order d and denoted by I(d) if it can be expressed as:

$$(1 - L)^d x_t = u_t \qquad t = 1, 2, \ldots, \qquad (3)$$

where L is the lag operator and u_t is I(0) or short memory. Note that the polynomial in L on the left-hand side of (3) can be expanded in terms of its Binomial representation such that, for any real d,

$$(1 - L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \cdots \qquad (4)$$

and thus equation (3) becomes:

$$x_t = d x_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \cdots + u_t \qquad (5)$$

According to equation (5), if the differencing parameter d is an integer, x_t depends only on a finite number of previous observations; however, if it is a fractional value, the series will depend on its entire past history. Moreover, the higher d is, the higher will be the association between observations, and therefore this parameter can be interpreted as a measure of the degree of persistence (dependence) in the series. In this context, if d is smaller than 0.5, x_t is covariance stationary; however, $d \geq 0.5$ implies non-stationarity, although if $d<1$ the process is still mean-reverting, with shocks having transitory effects which decay to zero hyperbolically; in addition, x_t in (3) admits an infinite MA representation; finally, if $d \geq 1$, the process is explosive and shocks have permanent effects.

In this paper we estimate the differencing parameter d using a version of the method developed by Robinson (1994) and widely used in empirical applications of fractional integration (Gil-Alana and Robinson, 1997), which is essentially a frequency domain version of the likelihood function. This approach has numerous advantages over others. First, it is based on testing the null hypothesis H_o: $d = d_o$ in (3) for any real value d_o, including those outside the stationarity region ($d_o \geq 0.5$); second, it has a standard N(0,1) asymptotic distribution, and this behaviour holds whether or not deterministic terms are included in the model; third, it also allows for weak autocorrelation in the error term; fourth, it is the most efficient method in the Pitman sense against local departures from the null.

Our aim is to estimate d and thus assess the persistence not only of the debt/GDP ratio itself but also of each of its components appearing in the government budget constraint (ECB, 2011; Bouabdallah el al., 2017; Willems and Zettlemeyer, 2022, etc.):

$$b_t = \frac{1+r_t}{1+g_t} b_{t-1} - pb_t + sf_t \tag{6}$$

where b_t is the gross public debt-to-GDP ratio at time t, as a percentage of GDP, r_t is the real long-term government bond yield, g_t is the real GDP growth rate, pb_t is the government primary balance-to-GDP ratio, and sf_t is the stock-flow adjustment-to-GDP ratio, which includes financial transactions or other factors affecting the outstanding stock of debt (such as exchange rate changes or other statistical adjustments). To test for debt sustainability (namely, for the stationarity of the first-differenced debt series), one can use the following equation:

$$\Delta b_t = \frac{r_t - g_t}{1+g_t} b_{t-1} - pb_t + sf_t \qquad (7)$$

Note that the first term $\frac{1+r_t}{1+g_t} b_{t-1}$ in equation (7) is usually called the "snowball" effect or the interest rate growth rate differential; it shows that the debt ratio tends to rise (decline) if the GDP growth rate is lower (higher) than the interest rate paid on government debt (ECB, 2009), and therefore it captures the joint impact of interest payments and real GDP growth on the outstanding stock of debt relative to GDP. It is therefore possible for the debt/GDP ratio to be stable even in the presence of an increasing primary deficit and/or other factors if this offset by the snowball effect.

If the other financial transactions term is zero (sf_t), the debt ratio will stabilise when $(r_t - g_t)b_{t-1} \approx pb_t$. Thus, if $r_t > g_t$ (which has often been the case in the past) a primary surplus is needed to prevent the debt burden from increasing and an ever-larger surplus is needed to reduce it (Bouabdallah el al., 2017). However, in recent times $r_t < g_t$, which has led to governments to being less concerned about debt increases since in such a scenario solvency is still achieved (Blanchard, 2019). In this situation $(r_t < g_t)$, a stable solution for (7) might be $b_t = \frac{1+g_t}{r_t - g_t} pb_t$ (Willems and Zettlemeyer, 2022), implying that, for b_t to stabilize at a constant debt ratio, the primary balance must be in deficit and generating a "free lunch". Whilst previous papers such as Martin-Valmayor et al. (2024)

only analysed the primary balance-snowball relationship by carrying out Granger (1969) causality tests, the present one sheds more light on the issues of interest by applying fractional integration methods to measure the persistence of each of the terms of the government budget constraint, and then also assessing debt sustainability as initially suggested by Trehan and Walsh (1991).

4. Empirical Analysis

4.1 Data Sources and Description

The data source for all series is the Historical Public Debt Database (1800-2022) from the public IMF e-library (IMF, 2024b). The frequency is annual, and the sample period starts in 1831 for the UK, in 1862 for Italy and in 1881 for France, Germany, and the US (these being the five countries being analysed), and it ends in all cases in 2022. Additional G7 countries are not included in the analysis owing to the lack of bond yield series spanning a long time period. The IMF series used for the analysis are gross public debt-to-GDP ratio at time t, as a percentage of GDP (b_t); the real long-term government bond yield (r_t); the real GDP growth rate (g_t); and the government primary balance-to-GDP ratio (pb_t). The snowball terms were calculated using the expression $\frac{r_t - g_t}{1+g_t} b_{t-1}$ appearing in equation (7), while the stock-flow adjustment-to-GDP ratio was computed, from equation (6), as $sf_t = b_t - \frac{1+r_t}{1+g_t} b_{t-1} + pb_t$. Figure 1 displays the debt/GDP ratio for the selected countries. Different trajectories can be observed in the earlier part of the sample, while similar patterns are noticeable in the following period, including the impact of the two world wars which led to a sharp increase of this ratio in all countries. Table 1 reports descriptive statistics for all the series. It can be seen that the snowball and other stock flow components are the most volatile, while bond yields and the debt/GDP ratios exhibit the lowest volatility. There is also evidence of positive or negative skewness in most cases, the distribution being symmetrical only in the case of GDP growth in the US and Italy, and of the debt/GDP ratio in the UK and Italy. Further, most series exhibit excess kurtosis (k>3), the only exceptions being the primary balance in France and the debt/GDP ratio in the UK, Italy and Germany.

TABLE 1 AND FIGURE 1 ABOUT HERE

Figure 2 displays for each country the evolution over time of each of the series entering the budget constraint. It can be seen that war periods are characterised by high volatility and major stock-flow adjustments. This is consistent with the evidence provided by Martin-Valmayor et al. (2024) that the debt/GDP series exhibit structural breaks, especially during such periods.

FIGURE 2 ABOUT HERE

4.2 Persistence Analysis

For each of the series we specify a fractional integration model that allows for deterministic terms such as a constant and/or a linear time trend. In particular, the estimated model is the following one:

$$y_t = \gamma + \delta t + x_t, \qquad (1-L)^d x_t = u_t, \qquad (8)$$

where y_t is the observed time series, γ and δ are unknown parameters (specifically, the intercept and the coefficient on a linear time trend), and d is a real number corresponding to the order of integration of the series. Positive values imply that the series exhibits long memory, past observations having a greater impact the greater the value of d is, with mean reversion occurring if this parameter is below 1. The error term u_t is assumed to follow the exponential spectral model of Bloomfield (1973) that approximates AR structures in a non-parametric way.

The estimates are reported in Table 2. As is standard in the literature, we consider three possible model specifications, namely with i) no regressors (see column 2), ii) an intercept only (column 3), and iii) an intercept as well as a linear time trend (column 4). The model displayed in bold in each case is the one selected on the basis of the statistical significance of the estimated coefficients. Table 3 summarises the best estimates for the differencing parameter.[1]

TABLES 2 AND 3 ABOUT HERE

The results suggest that the debt/GDP ratio is a highly persistent I(1) series in all cases – in other words, no mean reversion occurs and shocks have permanent effects, as also found by previous studies using US post-WWII data such as Trehan and Walsh (1991). Concerning the individual components, we find stationary behaviour (0<d<0.5) in all cases except Germany for the snowball effect $\left(\frac{r_t-g_t}{1+g_t}b_{t-1}\right)$, and other financial components (sf_t) are also characterised by I(0) stationarity. By contrast, in the case of the primary balance (pb_t) we find low persistence with mean reversion in Germany and in the US, but much higher values of d for the other countries (0.5<d<1), with d=1 being within the confidence interval in the case of Italy.

Concerning the components of the snowball term $\frac{r_t-g_t}{1+g_t}b_{t-1}$, all bond yield series (r_t) are characterised by fractional integration (I(d) behaviour), with larger values of d being estimated for the US, Italy and Germany (0.5<d<1). We also find mean reversion patterns (0.5<d<1) in the GDP growth series (g_t), except for the US and the UK, where the stationarity I(0) hypothesis cannot be rejected. The differential between these two

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[1] Since the starting date is not the same for all countries as a robustness check we have also re-estimated the models starting in 1881 in all cases. The results (not reported to save space but available upon request) are consistent with those discussed in the main text in terms of the order of integration of the series.

series, $(r_t - g_t)$, also appears to be I(0) stationary in all cases, except for Germany where anti-persistence (d < 0) occurs, the entire confidence interval being below 0.

These results imply that, although the debt/GDP ratio is persistent in all cases, the reasons for this behaviour vary across countries. For instance, in Italy and France the primary balance is persistent (0.5<d<1), while both the snowball term and other financial transactions exhibit I(0) behaviour. Therefore, in these two countries (which have the highest debt/GDP ratio of the five under consideration) debt persistence is linked to that of the fiscal deficit. On the other hand, in countries such as Germany characterised by fiscal prudence we observe I(0) and mean-reverting behaviour of the primary balance, and anti-persistent values of d (d<0) for the (r-g) differential as well as highly persistent bond yields. Note that Germany has the lowest debt/GDP ratio of the countries in our sample, and because of its solvency and low deficits it can offer lower yields and still attract funds. The UK and the US appear to sit in the middle. Of these two countries the former is closer to France and Italy, having a primary balance which is not highly persistent, while in the US both the primary balance and the (r-g) differential exhibit stationary I(0) behaviour.

A correlation analysis produces results which are broadly consistent with the previous evidence (see Table 4). In France and Italy, the debt/GDP ratio is mainly correlated to the primary balance, whilst in Germany there is a large negative correlation with bond yields, and in the UK and the US almost all correlations are negative, those with the primary deficit being the biggest in absolute terms.

TABLE 4 ABOUT HERE

4.3 Debt Sustainability Tests

It is important also to address directly the issue of debt sustainability. In a useful paper, Chalk and Hemming (2000) reviewed early government solvency tests based on historical data. Previously, Hamilton and Flavin (1986) had suggested that a sufficient (but not necessary) condition for the stationarity of debt is that the primary balance be stationary, which holds only for Germany and the US according to our results. Subsequent paper such as Hansen et al. (1987), Campbell and Shiller (1987) and Trehan and Walsh (1988) introduced other sustainability conditions based on cointegration between net-of-interest expenditures, revenues, interest payments, and the outstanding stock of debt. Trehan and Walsh (1991) relaxed the requirement that expenditures and revenues be difference-stationary and provided sufficient conditions for sustainability. Specifically, if interest rates are constant, and debt as well as the primary balance are integrated of order 1, a sufficient condition for sustainability is that the latter two variables should be cointegrated. Instead, if interest rates are not constant, as long as the expected real rate of interest is positive, debt sustainability holds if the deficit inclusive of interest payments is stationary.

In our case, as the other stock-flow adjustments sf_t are stationary and $\frac{g_t}{1+g_t} \approx 0$; if follows that $\Delta b_t = \frac{r_t - g_t}{1+g_t} b_{t-1} - pb_t + sf_t$ will be stationary if $r_t b_{t-1} - pb_t$, namely the difference between interest payments and the primary balance, is I(0) stationary. As the debt/GDP series often exhibit structural breaks, especially during war periods (Martin-Valmayor et al., 2024), we estimate the model for both the full sample and the period starting in 1950. Table 5 reports the estimates of the differencing parameter d (and the 95% confidence interval) for the three series under investigation, i.e., bond yields, the primary balance and the debt sustainability condition (as in Trehan and Walsh, 1991) using the three specifications mentioned above. It can be seen that mean reversion (d<1) occurs in all cases for the Trehan and Walsh (1991) sustainability condition, which implies solvency.

TABLE 5 ABOUT HERE

The estimated values of d are in the interval (0.22<d<0.52); surprisingly, the lowest ones are found for France (d=0.22) and Italy (d=0.23), even though the primary balance in the case of Italy is I(1). A possible explanation is that the debt/GDP ratio in France (95%) and Italy (98%) at the beginning of the sample period (1881) is similar to today's levels. The picture does not change significantly if only data for the period starting in 1950 are analysed, though the interval for d becomes narrower (0.40 < d < 0.54) and there is evidence of slow mean reverting properties in all countries. However, during that period the primary balance worsened in all countries, Germany being the only one which kept a significant primary surplus, whilst over time the UK moved from a surplus to a balanced budget. The other countries (France, Italy and the US) are characterised by persistent deficits. Even though the Trehan and Walsh (1991) sustainability test indicates that the difference between interest payments and primary surplus is an I(d) series with slow mean reversion, the effects of those persistent deficits (given the budget constraint relationship $\Delta b_t = \frac{r_t - g_t}{1+g_t} b_{t-1} - pb_t$) would require an average GDP growth rate (g_t) of 1.4% in France, 1.98% in the US and 2.59% in Italy to keep the debt/GDP ratio on a sustainable path in the long term.

Some studies argue that the persistence of the primary deficit is a consequence of the way governments tend to respond to external shocks **(**Antonini et al., 2013): they will typically adopt expansionary fiscal policies which will generate deficits, and such policies will not be subsequently reversed, since this would require unpopular measures that could affect the outcome of future elections; thus governments do not make symmetrical corrections to generate primary surpluses (Beqiraj et al., 2018), especially if there is no pressure from bond holders or international organisations (Martin-Valmayor et al. 2024). This raises questions about debt sustainability, especially in the presence of relatively high yields as in the current economic environment, regardless of the results of the Trehan and Walsh (1991), with Japan being a special case, since it has a very large debt/GDP ratio (2.5) combined with a negative interest rate policy. As pointed out by Willems and Zettelmeyer (2022), the degree of credibility of the central bank is crucial in this context. Whenever government borrowing costs are below the growth rate of the economy (r < g) debt sustainability does not require future primary surpluses and there is an apparent "free lunch".

To obtain additional evidence on debt sustainability and the possible role played by GDP growth, we extend the Trehan and Walsh (1991) analysis further; specifically, we use the same fractional integration methods as before to examine the degree of persistence of the budget constraint relationship, $\Delta b_t = \frac{r_t - g_t}{1+g_t} b_{t-1} - pb_t$, without including sf_t, this being an I(0) series. These results are displayed in Table 6; in general, the estimates of the differencing parameter d are now lower, regardless of whether one considers the full sample (0.17<d<0.42) or the period after 1950 (0.20<d<0.41). This evidence of mean reversion implies that debt is on a sustainable path in all cases, possibly as a result of the positive effects of a relatively high GDP growth rate.

TABLE 6 ABOUT HERE

5. Conclusions

This paper analyses the persistence and mean reversion properties of sovereign debt and its individual components appearing in the budget constraint equation for the US and the largest European economies. Specifically, fractional integration methods are applied to long runs of annual data starting in 1831 for the UK and the US, in 1862 for Italy and in

1881 for France and Germany, and ending in all cases in 2022. The chosen approach is more general and flexible than the standard one based on the classical I(0) versus I(1) dichotomy, and thus it encompasses a wide range of stochastic processes, the unit root case being one of them.

The empirical results provide evidence of a high degree of persistence in all debt/GDP series, which appear to be I(1), consistently with the findings of previous studies such as Trehan and Walsh (1991), Antonini et al. (2013), Cuestas et al. (2014), Chen et al. (2018) and Caporale et al. (2021). However, cross-country differences emerge when analysing the properties of the individual components of debt. Specifically, in countries with a relatively large debt/GDP ratio, such as France and Italy, the primary balance is more persistent (0.5<d<1) than the snowball term $\left(\frac{r-g}{1+g}\right) b_{t-1}$, while in those with lower debt, such as Germany and the US, it is stationary but bond yields are persistent (0.5<d<1). In all cases, the other financial transactions series is stationary and peaking in war periods, while GDP growth exhibits low persistence and mean reversion (0<d<0.5). Therefore, it appears that persistence in the debt/GDP ratio reflects different factors depending on the country being investigated and its degree of fiscal prudence.

Regarding the issue of debt sustainability, the present paper extends the analysis of the seminal work by Trehan and Walsh (1991) in two ways. First, as already mentioned, it uses a fractional integration approach which is more general than the unit root testing carried out in that study as well as in several others on this topic. Second, instead of focusing only on the post-WWII period in the US it examines long time spans from the 19th century till the present days for the US as well as the main European economies. Following Trehan and Walsh (1991), to assess sustainability we analyse the stochastic properties of the differential between interest payments and primary deficits $(r_t b - pb_t)$ and find that more heavily indebted countries such as Italy and France are characterised by stationarity of this series, which implies sustainability. However, this seemingly puzzling result is likely to reflect the fact that France and Italy already had very high debt/GDP ratios, comparable to the current ones, at the beginning of the sample period. Therefore, we have re-examined this issue using only data from 1950; in this case we obtain more homogeneus results for all countries ($0.40<d<0.54$), all of them still appearing to be on a sustainable path. This conclusion is confirmed by further persistence tests based on the budget constraint as a whole, which possibly reflects relatively high GDP growth rates in the sample countries over the period under investigation.

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Table 1. Descriptive statistics for the terms of the budget constraint equation in each country under investigation

FRANCE	**Bond yield** (r_t)	**GDP growth** (g_t)	**Snowball** $\frac{r_t-g_t}{1+g_t}b_{t-1}$	**Primary balance** (pb_t)	**Other stock flow** (sf_t)	**DEBT/GDP** (b_t)
Mean	1.46	2.14	-0.14	0.19	-0.41	79.97
Median	1.92	2.43	-0.17	0.53	-0.46	79.73
Maximum	20.32	52.05	46.81	5.67	34.09	237.04
Minimum	-34.91	-21.11	-63.76	-8.25	-48.03	14.39
Std.Dev.	7.55	6.59	13.59	3.07	11.38	48.93
Std.Dev/Mean	5.16	3.07	-97.30	16.44	-27.54	0.61
Skewness	-1.01	1.53	-0.91	-0.76	-0.73	0.76
Kurtosis	7.17	19.88	10.13	2.95	7.83	3.47
First obs.	1881	1821	1881	1880	1881	1880
No. of obs.	142	202	142	143	142	143
GERMANY	**Bond yield** (r_t)	**GDP growth** (g_t)	**Snowball** $\frac{r_t-g_t}{1+g_t}b_{t-1}$	**Primary balance** (pb_t)	**Other stock flow** (sf_t)	**DEBT/GDP** (b_t)
Mean	2.96	2.54	0.26	1.00	-1.02	36.43
Median	3.16	2.82	-0.11	1.26	-1.12	35.83
Maximum	21.23	18.90	26.34	4.34	26.10	81.99
Minimum	-7.51	-52.59	-6.28	-5.98	-10.57	9.73
Std.Dev.	3.43	6.88	2.95	1.54	3.38	17.74
Std.Dev/Mean	1.16	2.71	11.23	1.55	-3.30	0.49
Skewness	1.27	-3.69	5.26	-1.28	3.79	0.76
Kurtosis	9.01	28.93	45.16	6.13	32.12	2.72
First obs.	1881	1851	1881	1880	1881	1880
No. of obs.	142	172	142	143	142	143

ITALY	Bond yield (r_t)	GDP growth (g_t)	Snowball $\frac{r_t-g_t}{1+g_t}b_{t-1}$	Primary balance(pb_t)	Other stock flow (sf_t)	DEBT/GDP (b_t)
Mean	1.87	2.29	0.42	-0.92	0.43	87.87
Median	3.00	2.33	0.81	0.95	-0.37	91.76
Maximum	24.01	31.00	23.07	6.55	54.12	159.72
Minimum	-36.23	-21.71	-32.13	-34.05	-49.07	24.21
Std.Dev.	7.40	5.49	7.77	6.65	9.32	33.76
Std.Dev/Mean	3.95	2.40	18.69	-7.24	21.47	0.38
Skewness	-2.02	-0.22	-1.05	-2.42	0.69	-0.28
Kurtosis	11.00	10.57	7.04	9.81	18.38	2.24
First obs.	1861	1861	1861	1862	1861	1861
Observations	161	161	161	161	162	162
UK	**Bond yield (r_t)**	**GDP growth (g_t)**	**Snowball $\frac{r_t-g_t}{1+g_t}b_{t-1}$**	**Primary balance(pb_t)**	**Other stock flow (sf_t)**	**DEBT/GDP (b_t)**
Mean	2.84	1.98	-0.09	0.59	-0.61	111.61
Median	2.57	2.22	0.02	2.21	0.11	97.85
Maximum	41.45	10.01	12.90	11.39	22.07	269.80
Minimum	-16.41	-11.03	-13.02	-47.67	-172.30	27.27
Std.Dev.	6.56	3.04	2.42	8.57	13.43	65.84
Std.Dev/Mean	2.31	1.54	-25.48	14.50	-22.17	0.59
Skewness	1.06	-1.04	-0.85	-3.05	-10.87	0.50
Kurtosis	9.53	6.26	14.17	14.40	139.86	2.08
First obs.	1801	1831	1831	1830	1830	1800
No. of obs.	222	192	192	193	193	223

USA	**Bond yield** (r_t)	**GDP growth** (g_t)	**Snowball** $\frac{r_t - g_t}{1+g_t} b_{t-1}$	**Primary balance**(pb_t)	**Other stock flow** (sf_t)	**DEBT/GDP** (b_t)
Mean	2.12	3.65	-0.73	-0.58	-0.35	33.75
Median	2.71	3.49	-0.30	0.21	0.00	22.95
Maximum	15.84	19.73	7.08	6.56	6.74	133.50
Minimum	-13.38	-13.17	-13.63	-26.46	-10.63	0.00
Std.Dev.	4.34	4.91	2.98	3.88	1.89	32.28
Std.Dev/Mean	2.05	1.35	-4.05	-6.73	-5.38	0.96
Skewness	-0.74	0.11	-1.54	-3.37	-1.94	1.01
Kurtosis	6.40	4.96	8.25	18.06	12.38	3.21
First obs.	1,881	1,801	1,881	1,800	1,881	1,800
No. of obs.	142	222	142	223	142	223

Note: The data source for all series is the Historical Public Debt Database (1800-2022) from the public IMF e-library (IMF, 2024b). The frequency is annual, and the sample period starts in 1831 for the UK, in 1862 for Italy and in 1881 for France, Germany, and the US.

Table 2: Estimates of the differencing parameter d for the whole sample (those from the selected model are in bold)

Series	No terms	An intercept	An intercept and a linear time trend
FRANCE [1881 – 2022]			
Debt/GDP	1.23 (1.01, 1.54)	**1.22 (1.00, 1.60)**	1.22 (1.00, 1.60)
Bond Yield (r)	**0.37 (0.25, 0.53)**	0.37 (0.24, 0.53)	0.37 (0.25, 0.53)
GDP Growth (g)	0.07 (-0.02, 0.20)	**0.07 (-0.02, 0.21)**	0.07 (-0.04, 0.20)
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	0.69 (0.52, 0.93)	**-0.02 (-0.21, 0.26)**	-0.03 (-0.22, 0.34)
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	**0.29 (0.10, 0.51)**	0.29 (0.10, 0.51)	0.29 (0.10, 0.52)
Primary Balance	0.67 (0.50, 0.90)	0.66 (0.48, 0.93)	**0.68 (0.49, 0.93)**
Others	**0.07 (-0.16, 0.39)**	0.07 (-0.15, 0.38)	0.07 (-0.16, 0.38)
ITALY [1862 – 2022]			
Series	No terms	An intercept	An intercept and a linear time trend
Debt/GDP	0.97 (0.82, 1.16)	**0.89 (0.74, 1.09)**	0.90 (0.75, 1.10)
Bond Yield (r)	**0.55 (0.42, 0.71)**	0.54 (0.42, 0.71)	0.55 (0.42, 0.71)
GDP Growth (g)	0.14 (0.03, 0.29)	**0.14 (0.04, 0.29)**	0.14 (0.03, 0.29)
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	0.75 (0.60, 0.94)	**0.09 (-0.06, 0.90)**	0.12 (-0.06, 0.91)
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	**0.22 (0.13, 0.33)**	0.21 (0.13, 0.33)	0.22 (0.13, 0.34)
Primary Balance	**0.67 (0.41, 1.00)**	0.66 (0.41, 0.99)	0.66 (0.41, 0.99)
Others	**0.07 (-0.07, 0.27)**	0.07 (-0.07, 0.27)	0.07 (-0.08, 0.28)
GERMANY [1881 – 2022]			
Series	No terms	An intercept	An intercept and a linear time trend
Debt/GDP	1.09 (0.90, 1.36)	**1.02 (0.87, 1.22)**	1.02 (0.86, 1.23)
Bond Yield (r)	**0.65 (0.50, 0.85)**	0.66 (0.51, 0.86)	0.67 (0.52, 0.86)
GDP Growth (g)	0.17 (0.02, 0.39)	**0.17 (0.01, 0.39)**	0.17 (0.02, 0.39)
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	0.57 (0.38, 0.78)	**-0.43 (-0.60, -0.17)**	-0.43 (-0.61, -0.17)
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	**0.13 (-0.03, 0.36)**	0.13 (-0.03, 0.36)	0.14 (-0.02, 0.37)
Primary Balance	0.24 (0.03, 0.50)	0.19 (0.03, 0.46)	**0.16 (-0.05, 0.49)**
Others	0.06 (-0.13, 0.29)	**0.06 (-0.10, 0.27)**	0.05 (-0.10, 0.28)

UNITED KINGDOM [1831 – 2022]			
Series	No terms	An intercept	An intercept and a linear time trend
Debt/GDP	1.44 (1.38, 1.50)	1.44 (1.38, 1.50)	**1.44 (1.38, 1.50)**
Bond Yield (r)	0.27 (0.16, 0.42)	0.23 (0.14, 0.39)	**0.22 (0.11, 0.38)**
GDP Growth (g)	0.11 (-0.05, 0.33)	**0.10 (-0.05, 0.31)**	0.10 (-0.05, 0.33)
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	0.50 (0.49, 0.50)	0.27 (0.17, 0.36)	**0.25 (0.16, 0.35)**
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	0.26 (0.18, 0.34)	**0.26 (0.17, 0.34)**	0.24 (0.16, 0.32)
Primary Balance	0.44 (0.15, 0.92)	0.43 (0.15, 0.91)	**0.41 (0.11, 0.91)**
Others	**0.26 (0.24, 0.28)**	0.25 (0.20, 0.30)	0.24 (0.19, 0.29)
UNITED STATES [1831 – 2022]			
Series	No terms	An intercept	An intercept and a linear time trend
Debt/GDP	0.86 (0.66, 1.13)	0.87 (0.68, 1.16)	**0.84 (0.65, 1.16)**
Bond Yield (r)	**0.60 (0.42, 0.86)**	0.60 (0.42, 0.87)	0.61 (0.44, 0.87)
GDP Growth (g)	0.00 (-0.10, 0.17)	**0.00 (-0.10, 0.15)**	-0.01 (-0.13, 0.14)
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	0.01 (-0.04, 0.06)	**-0.06 (-0.25, 0.23)**	-0.05 (-0.25, 0.25)
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	**0.22 (0.10, 0.39)**	0.22 (0.10, 0.39)	0.23 (0.11, 0.31)
Primary Balance	0.01 (-0.20, 0.35)	0.02 (-0.22, 0.33)	**0.01 (-0.22, 0.34)**
Others	0.01 (-0.15, 0.26)	**0.01 (-0.16, 0.25)**	-0.01 (-0.18, 0.28)

Note: The values in bold refer to the estimates of d (and their 95% confidence bands) for the selected specification based on the statistical (in)significance of the deterministic terms.

Table 3: Summary of the estimates of the differencing parameter d for the whole sample from the selected specifications

FRANCE [1881 – 2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Debt/GDP	I(1)	1.22 (1.00, 1.60)	95.628 (10.55)	---
Bond Yield (r)	I(d)	0.37 (0.25, 0.53)	---	---
GDP Growth (g)	I(d)	0.22 (0.11, 0.38)	6.031 (3.15)	-0.027 (-1.88)
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	I(0)	-0.02 (-0.21, 0.26)	0.989 (105.18)	---
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	I(d)	0.29 (0.10, 0.51)	---	---
Primary Balance	I(d)	0.68 (0.49, 0.93)	3.815 (3.31)	-0.046 (-1.78)
Others	I(0)	0.07 (-0.16, 0.39)	---	---
ITALY [1862 – 2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Debt/GDP	I(1)	0.89 (0.74, 1.09)	43.867 (5.37)	---
Bond Yield (r)	I(d)	0.55 (0.42, 0.71)	---	---
GDP Growth (g)	I(d)	0.14 (0.04, 0.29)	2.173 (2.67)	
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	I(0)	0.09 (-0.06, 0.90)	1.001 (89.89)	---
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	I(d)	0.22 (0.13, 0.33)	---	---
Primary Balance	I(1)	0.67 (0.41, 1.00)	---	---
Others	I(0)	0.07 (-0.07, 0.27)	---	---
GERMANY [1881 – 2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Debt/GDP	I(1)	1.02 (0.87, 1.22)	29.683 (12.97)	---
Bond Yield	I(d)	0.65 (0.50, 0.85)	---	---
GDP Growth	I(d)	0.17 (0.01, 0.39)	---	
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	I(d<0)	-0.43 (-0.60, -0.17)	1.012 (90.42)	
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	I(0)	0.13 (-0.03, 0.36)	---	---
Primary Balance	I(0)	0.16 (-0.05, 0.49)	1.852 (5.12)	-0.012 (-2.81)
Others	I(0)	0.06 (-0.10, 0.27)	-1.061 (-3.14)	---

UNITED KINGDOM [1831 – 2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Debt/GDP	I(1)	1.44 (1.38, 1.50)	-0.127 (-0.01)	0.002 (0.04)
Bond Yield (r)	I(d)	0.22 (0.11, 0.38)	6.031 (3.15)	-0.027 (-1.88)
GDP Growth (g)	I(0)	0.10 (-0.05, 0.31)	---	---
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	I(d)	0.25 (0.16, 0.35)	-0.000 (-0.29)	-0.000 (-0.11)
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	I(d)	0.26 (0.17, 0.34)	-0.025 (-0.72)	---
Primary Balance	I(d)	0.41 (0.11, 0.91)	5.232 (2.48)	-0.042 (-2.12)
Others	I(d)	0.26 (0.24, 0.28)	---	---
UNITED STATES [1881 – 2022]				
Series		No terms	An intercept	An intercept and a linear time trend
Debt/GDP	I(1)	0.84 (0.65, 1.16)	14.689 (3.05)	0.719 (3.58)
Bond Yield	I(d)	0.60 (0.42, 0.86)	---	---
GDP Growth	I(0)	0.00 (-0.10, 0.15)	3.649 (11.709)	
$\left(\frac{1+r}{1+g}\right) \approx (r-g)$	I(0)	-0.06 (-0.25, 0.23)	0.991 (214.56)	---
Snowball: $\left(\frac{r-g}{1+g}\right) Debt_{t-1}$	I(d)	0.22 (0.10, 0.39)	---	---
BP	I(0)	0.01 (-0.22, 0.34)	0.129 (2.25)	-0.017 (-2.82)
Others	I(0)	0.01 (-0.16, 0.25)	-0.350 (-2.32)	---

Note: The values in parenthesis in the last two columns refer to the t-values of the intercept and the linear time trend.

Table 4: Debt/GDP correlation coefficients with the other variables

Debt/GDP correlation with	Bond Yield	GDP Growth	Primary Balance	Other stock flows
FRANCE	0.04	-0.12	0.28	-0.18
GERMANY	-0.27	-0.14	0.01	-0.17
ITALY	0.05	-0.41	0.22	-0.14
UK	0.01	-0.11	0.08	-0.07
US	-0.25	-0.08	-0.31	-0.21

Table 5: Estimation of the differencing parameter d for the debt sustainability condition (Trehan and Walsh, 1991), from the start of the sample and from 1950

Debt sustainability condition $(r_t b_{t-1} - pb_t)$ (Trehan and Walsh, 1991) – full sample				
FRANCE (1881-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	**0.22 (0.04, 0.43)**	0.22 (0.04, 0.43)	0.22 (0.03, 0.43)	I(d)
Interests: $r_t B_{t-1}$	**0.27 (0.20, 0.34)**	0.27 (0.20, 0.34)	0.27 (0.19, 0.34)	I(d)
Primary Balance: PB_t	0.67 (0.50, 0.90)	0.66 (0.48, 0.93)	**0.68 (0.49, 0.93)**	I(d)
GERMANY (1881-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	**0.52 (0.39, 0.70)**	0.51 (0.39, 0.70)	0.51 (0.38, 0.70)	I(d)
Interests: $r_t B_{t-1}$	**0.84 (0.74, 0.93)**	0.84 (0.73, 0.92)	0.71 (0.81, 0.91)	I(d)
Primary Balance: PB_t	0.24 (0.03, 0.50)	0.19 (0.03, 0.46)	**0.16 (-0.05, 0.49)**	I(0)
ITALY (1862-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	0.22 (0.11, 0.39)	**0.23 (0.11, 0.40)**	0.23 (0.11, 0.40)	I(d)
Interests: $r_t B_{t-1}$	**0.45 (0.40, 0.50)**	0.45 (0.39, 0.50)	0.45 (0.38, 0.51)	I(d)
Primary Balance: PB_t	**0.67 (0.41, 1.00)**	0.66 (0.41, 0.99)	0.66 (0.41, 0.99)	I(1)
UK (1831-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	**0.44 (0.37, 0.51)**	0.44 (0.37, 0.51)	0.44 (0.36, 0.51)	I(d)
Interests: $r_t B_{t-1}$	**0.36 (0.30, 0.42)**	0.34 (0.28, 0.41)	0.32 (0.26, 0.39)	I(d)
Primary Balance: PB_t	0.44 (0.15, 0.92)	0.43 (0.15, 0.91)	**0.41 (0.11, 0.91)**	I(d)
US (1881-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	**0.46 (0.40, 0.52)**	0.46 (0.39, 0.54)	0.46 (0.38, 0.53)	I(d)
Interests: $r_t B_{t-1}$	**0.46 (0.40, 0.52)**	0.46 (0.38, 0.53)	0.46 (0.38, 0.53)	I(d)
Primary Balance: PB_t	0.01 (-0.20, 0.35)	0.02 (-0.22, 0.33)	**0.01 (-0.22, 0.34)**	I(0)
Debt sustainability condition $(r_t b_{t-1} - pb_t)$ (Trehan and Walsh, 1991) – after 1950				
FRANCE (1950-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	**0.43 (0.29, 0.69)**	0.47 (0.31, 0.76)	0.46 (0.26, 0.76)	I(d)
Interests: $r_t B_{t-1}$	**0.48 (0.41, 0.56)**	0.48 (0.41, 0.56)	0.48 (0.40, 0.56)	I(d)
Primary Balance: PB_t	**0.47 (0.38, 0.57)**	0.47 (0.33, 0.60)	0.46 (0.33, 0.60)	I(d)
GERMANY (1950-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	

$(r_t B_{t-1} - PB_t)$	**0.45 (0.29, 0.71)**	0.45 (0.29, 0.71)	0.44 (0.30, 0.72)	I(d)
Interests: $r_t B_{t-1}$	0.49 (0.43, 0.54)	0.49 (0.43, 0.54)	**0.49 (0.44, 0.54)**	I(d)
Primary Balance: PB_t	**0.40 (0.21, 0.59)**	0.39 (0.19, 0.59)	0.38 (0.18, 0.58)	I(d)
ITALY (1950-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	0.51 (0.37, 0.71)	**0.54 (0.39, 0.81)**	0.55 (0.40, 0.82)	I(d)
Interests: $r_t B_{t-1}$	**0.49 (0.44, 0.54)**	0.49 (0.44, 0.54)	0.49 (0.44, 0.54)	I(d)
Primary Balance: PB_t	**0.49 (0.44, 0.53)**	0.49 (0.44, 0.54)	0.49 (0.44, 0.54)	I(d)
UK (1950-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	**0.40 (0.30, 0.50)**	0.40 (0.25, 0.55)	0.34 (0.13, 0.55)	I(d)
Interests: $r_t B_{t-1}$	**0.40 (0.30, 0.50)**	0.40 (0.26, 0.54)	0.40 (0.26, 0.54)	I(d)
Primary Balance: PB_t	**0.47 (0.32, 0.62)**	0.47 (0.32, 0.62)	0.44 (0.26, 0.62)	I(d)
US (1950-2022)	**No terms**	**An intercept**	**An intercept and a linear time trend**	
$(r_t B_{t-1} - PB_t)$	**0.44 (0.31, 0.57)**	0.43 (0.29, 0.56)	0.38 (0.15, 0.61)	I(d)
Interests: $r_t B_{t-1}$	0.47 (0.38, 0.56)	0.47 (0.38, 0.56)	**0.47 (0.38, 0.56)**	I(d)
Primary Balance: PB_t	**0.47 (0.38, 0.57)**	0.47 (0.36, 0.58)	0.45 (0.31, 0.58)	I(d)

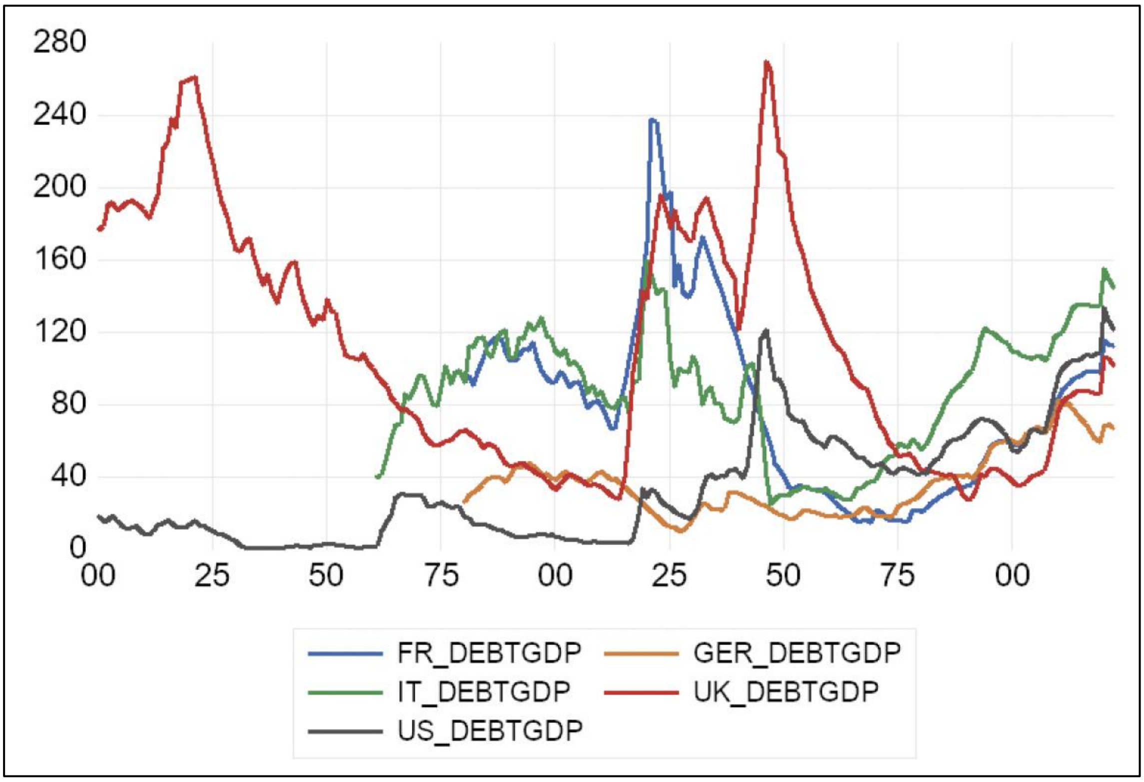
Note: The values in bold refer to the estimates of d (and their 95% confidence bands) for the selected specification based on the statistical (in)significance of the deterministic terms.

Table 6: Selected estimates of the differencing parameter d for the debt budget constraint relationship $\Delta b_t = \frac{r_t - g_t}{1+g_t} b_{t-1} - pb_t$

Budget constraint relationship $\Delta b_t = \frac{r_t - g_t}{1+g_t} b_{t-1} - pb_t$ **condition. Full sample**				
Series		No terms	An intercept	An intercept and a linear time trend
France (1881 – 2022)	I(d)	0.26 (0.19, 0.32)	---	---
Germany (1881 – 2022)	I(d)	0.24 (0.18, 0.30)	---	---
Italy (1862 – 2022)	I(d)	0.17 (0.11, 0.22)	---	---
UK (1831 – 2022)	I(d)	0.42 (0.34, 0.51)	-1.92 (0.17)	0.011 (0.183)
US (1881 – 2022)	I(d)	0.39 (0.32, 0.47)	---	---
Budget constraint relationship $\Delta b_t = \frac{r_t - g_t}{1+g_t} b_{t-1} - pb_t$ **condition. After 1950**				
Series		No terms	An intercept	An intercept and a linear time trend
France (1950 – 2022)	I(d)	0.20 (0.06, 0.34)	---	---
Germany (1950 – 2022)	I(d)	0.35 (0.13, 0.57)	---	---
Italy (1950 – 2022)	I(d)	0.20 (0.07, 0.33)	---	---
UK (1950 – 2022)	I(d)	0.33 (0.20, 0.47)	---	---
US (1950 – 2022)	I(d)	0.41 (0.33, 0.48)	---	---

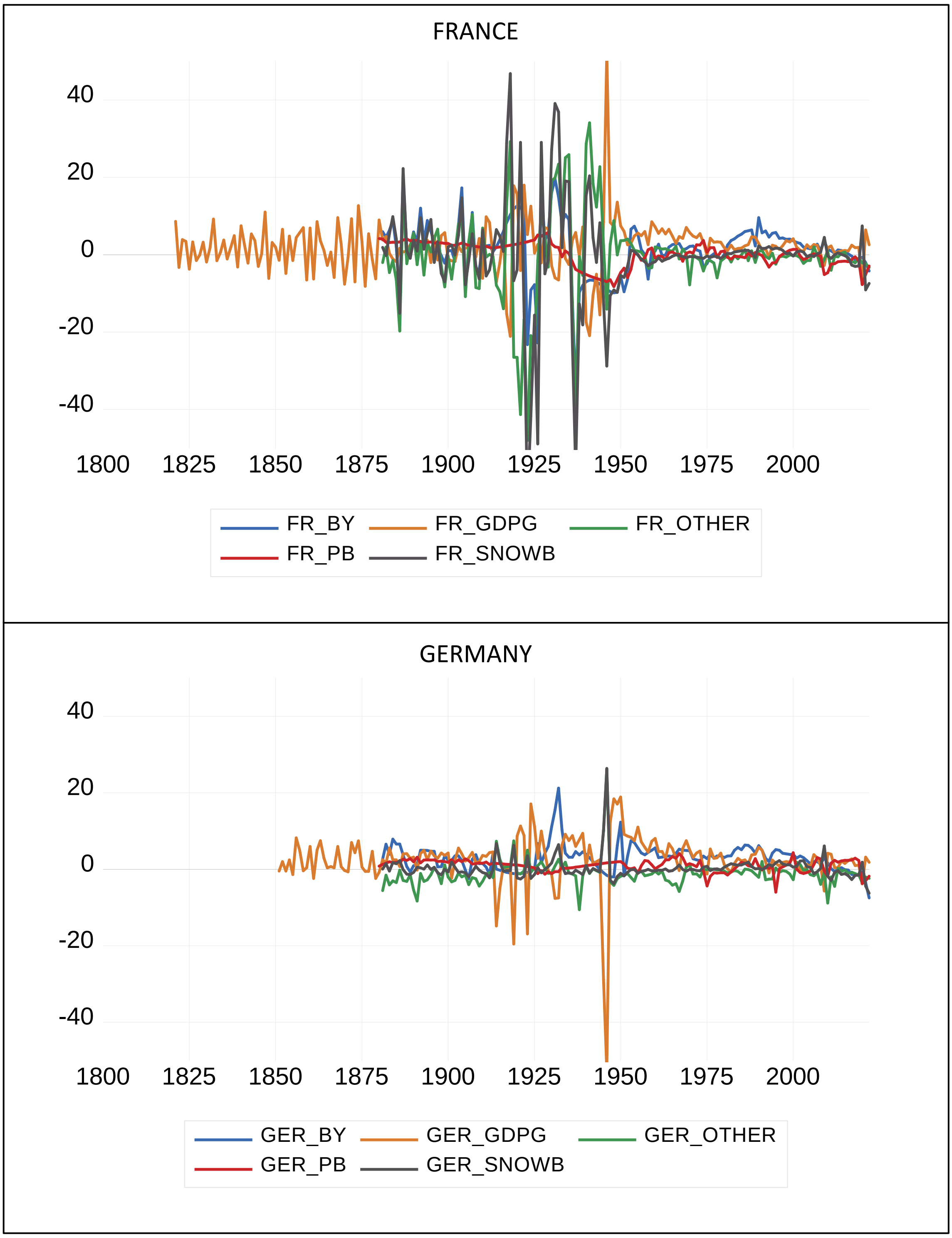
Note: The values in parenthesis in the last two columns refer to the t-values of the intercept and the linear time trend.

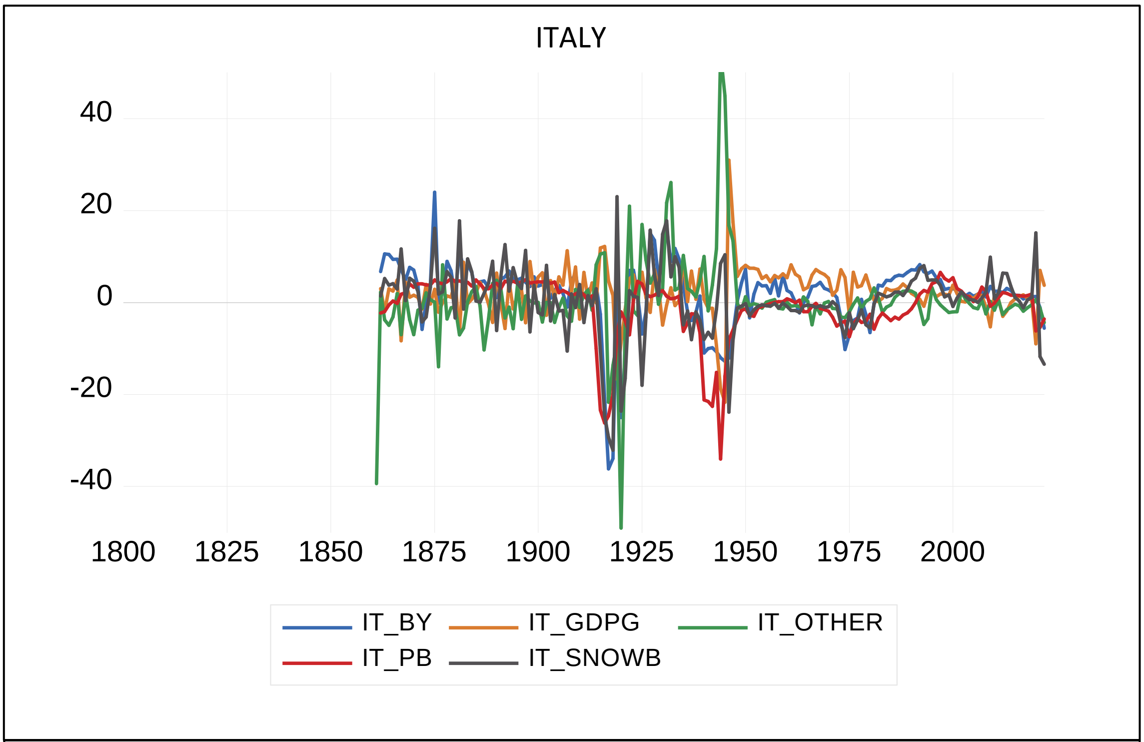
Figure 1. Debt/GDP series for the countries under investigation

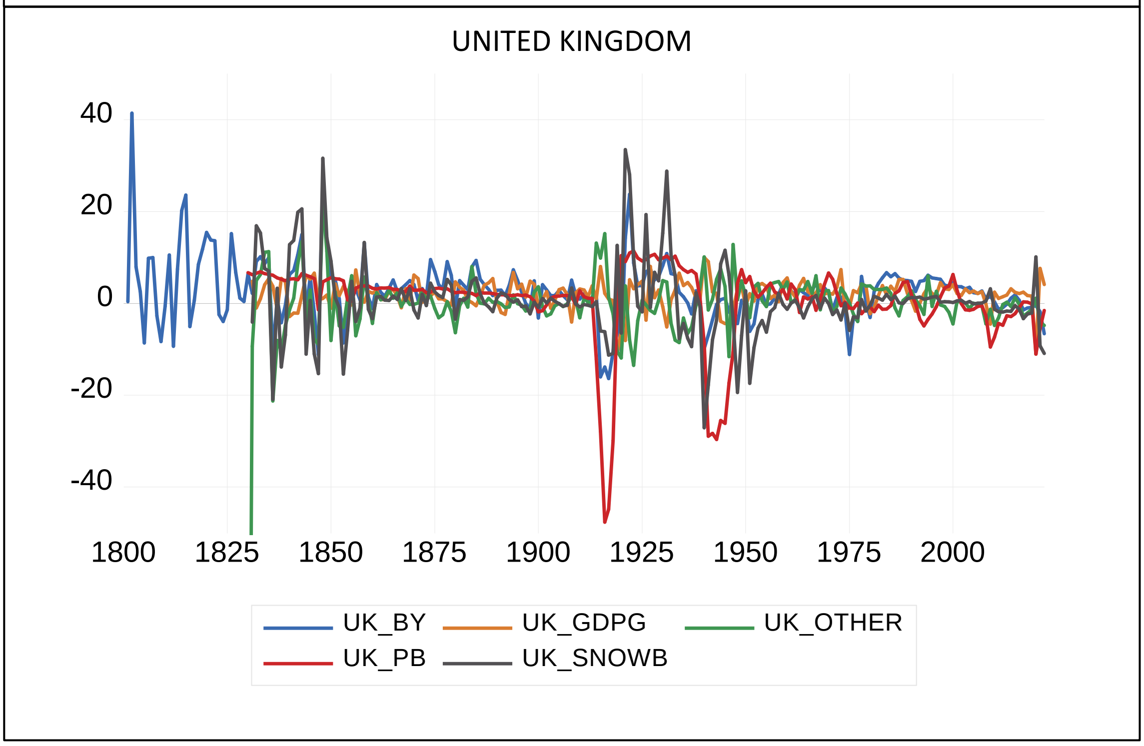


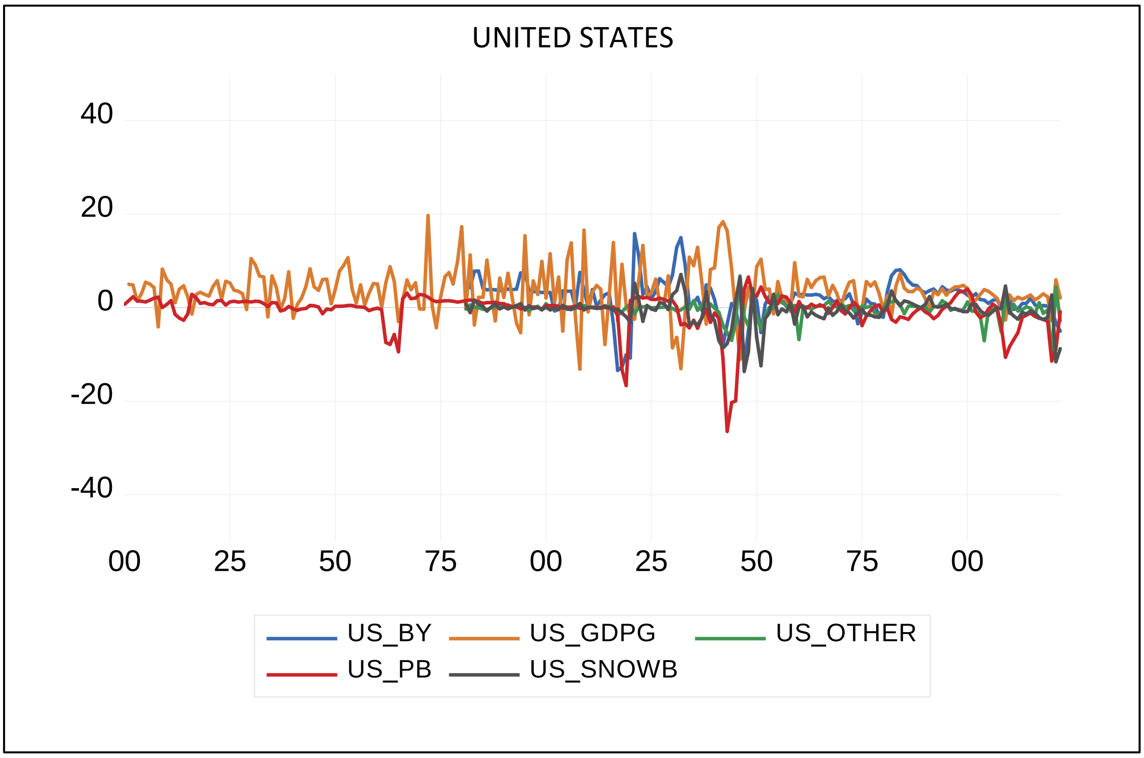
Note: The sample starts in 1800 for the UK and the US, 1860 for Italy and 1880 for Germany and France, and ends in 2022 in all cases. The data source is the Historical Public Debt Database (1800-2022) from the public IMF e-library (IMF, 2024b).

Figure 2. Debt/GDP components in the countries under investigation




ITALY
40
20
0
-20
-40
1800
1825
1850
1875
1900
1925
1950
1975
2000
IT_BY
IT_GDPG
IT_OTHER
IT_PB
IT_SNOWB


UNITED KINGDOM
40
20
0
-20
-40
1800
1825
1850
1875
1900
1925
1950
1975
2000
UK_BY
UK_GDPG
UK_OTHER
UK_PB
UK_SNOWB



Note: This figure plots for each country the variables appearing in the budget constraint: bond yields – BY; GDP growth – GDPG; primary balance – PB; snowball contribution – SNOWB; and the other stock-flow adjustments - OTHER).