Department of Economics and Finance

Working Paper No. 2417

Economics and Finance Working Paper Series

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November 2024

http://www.brunel.ac.uk/economics

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A FRACTIONAL INTEGRATION ANALYSIS

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November 2024

Abstract

This paper analyses monthly data on sea ice cover extent from January 1979 to April 2024 and on snow cover extent from January 1967 to April 2024 for various regions of the world using fractional integration methods. A statistically significant time trend is found when the errors in the regression model are incorrectly assumed to exhibit short memory. However, this evidence vanishes when the errors are allowed instead to follow an I(d) process with d different from zero and thus to be characterised by long memory (a well-known property of most climatological series) – in this case, the time trend becomes insignificant for all series. This implies that previous findings suggesting an irreversible decline in both sea ice and snow cover extent might be misleading because of an incorrect model specification.

Keywords: sea ice and snow cover extent, trends, fractional integration, long memory

JEL classification: C22, Q54

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* The second and third authors gratefully acknowledge financial support from an Internal Project of the Universidad Francisco de Vitoria. Luis A. Gil-Alana also acknowledges support from the MINEIC-AEI-FEDER PID2020-113691RB-I00 project from 'Ministerio de Economía, Industria y Competitividad' (MINEIC), `Agencia Estatal de Investigación' (AEI) Spain and `Fondo Europeo de Desarrollo Regional' (FEDER).

1. Introduction

Sea ice and snow cover extent both play a key role in the global climate system. The former is the area of ice that covers the Arctic Ocean and the Southern Ocean respectively at any given time. Its extent is crucial since sea ice reflects sunlight back into space as well as regulating ocean and air temperature, circulating ocean water, and preserving animal habitats. Snow instead is of essential importance for the water cycle. In particular, winter precipitation and spring and summer runoff both affect the water supply, energy production, and local climate. In addition, snow is the cause of geological hazards such as floods and avalanches. For these reasons, its cover extent is also considered a key climatological variable.

In recent decades there has been a growing concern that global warming is leading to an irreversible and sharp decline in both sea ice and snow cover extent, with severe damaging consequences for the planet Earth (see, e.g., Frei et al., 2012; Groisman et al., 1994a,b; Li et al., 2018). Existing studies invariable conclude that indeed those two series exhibit a negative and significant time trend (see, e.g., Déry and Brown, 2007; Young, 2023; Peng and Meier, 2018; Mudryk et al 2020a,b, 2021**)**. This evidence is based on standard modelling approaches which require the disturbances to be well behaved, namely not to be autocorrelated. However, it is well known that climatological series tend to exhibit long memory - see, e.g., the recent studies by Yuan et al. (2022), Zhu et al. (2023), and Gil-Alana and Carmona (2023). For this reason, the present paper adopts a modelling framework which allows the series of interest to be characterised by this property. More specifically, it analyses monthly data on sea ice cover extent from January 1979 to April 2024 and on snow cover extent from January 1967 to April 2024 for various regions of the world using a fractional integration framework based on the assumption that the errors are I(d) with d different from zero. This leads to the conclusion that the series being investigated are not in fact trended, in contrast to the evidence of a progressive decline in both sea ice and snow cover extent which is obtained using standard methods. The implication is that the findings reported by previous studies are a consequence of their adopting an inappropriate modelling approach not recognising that the series under examination in fact exhibit long-range dependence.

The paper is structured as follows: Section 2 provides a brief review of the existing literature relying on standard methods; Section 3 outlines the fractional integration model used in the current study; Section 4 describes the data and presents the empirical findings; Section 5 offers some concluding remarks.

2. Literature Review

Snow and sea ice cover play a crucial role in the global climate system (Chahine, 1992; Mudryk et al., 2020a), as changes in them can affect directly the Earth's surface albedo (which is defined as the fraction of incoming sunlight that is reflected back to the atmosphere) and hence surface energy (Frei et al., 2012; Groisman et al., 1994a,b; Li et al., 2018). Specifically, the sea-ice cover acts as a thermodynamic and mechanical insulator that dampens oceanic heat loss and wind-driven mechanical mixing (Sturm and Massom, 2009). Similarly, owing to its low heat conductivity, snow also effectively insulates the underlying soil, with significant effects on deep soil temperatures and permafrost extent (Zhang, 2005; Lawrence and Slater, 2010; Gouttevin et al., 2012). Also, snow cover melting has positive effects on available fresh water, vegetation, biogeochemical activity, and terrestrial and aquatic ecosystems (Callaghan et al., 2011; Marti et al., 2017). Therefore, changes in the snow and sea ice cover extent can modify the physical and biological processes that affect the climate system, both directly and indirectly (Callaghan et al., 2011; Mudryk et al 2020a,b, 2021).

The average maximum winter global sea ice extent which has been observed from 1966 to the present is approximately 19×10^6 km^2 (NOAA, 2024), while the average maximum winter snow extent in the Northern Hemisphere[1] is approximately 47×10^6 km^2 (Estilow et al., 2015; NOAA, 2024**.** Several studies have analysed the relationship between snow and sea ice cover extent on one side and Earth's albedo and surface energy on the other, and shown that when the former declines the Earth's albedo decreases and the Earth's surface warms as a result of more energy being absorbed (Kukla and Kukla, 1974; Groisman et al., 1994b).

A decline in the snow and the sea ice cover extent has been observed in many parts of the world (Brown et al., 2010; Najafi et al., 2016). In particular, it appears that the depletion of the Arctic sea ice cover, especially in the summer, has accelerated since satellite-based measurements became available in the late 1970s (Parkinson et al., 1999). Such measurements have helped the scientific community to monitor the evolution over time of both the snow and the sea ice cover extent. Multiple types of sensors from a wide range of missions have been used to obtain time series for both spanning long time periods, and considerable effort has been made to measure temporal and regional variability of the sea ice cover extent in the Arctic (Peng and Meier, 2017). Since the late 1970s, satellite measurements have recorded a decrease of 10–15% per decade in the Arctic annual minimum sea ice cover extent (Comiso and Nishio, 2008; Cavalieri and Parkinson, 2012). According to Peng et al (2017), Arctic ice has been declining at a faster rate from 1997 to 2016, compared with the average decline over the previous 20 years (1978–97). On the other hand, climate projections indicate that in the future the snow cover extent will continue to decrease at the same rate of about 0.8×10^6 km^2 per decade observed in the Northern Hemisphere (NH) since 1970 (Brown and Robinson, 2011), thus

[1] No snow extent data are available for the Southern Hemisphere from NOAA (2024).

contributing to warmer temperatures (Brown et al., 2010). However, there has been considerable intra-seasonal variability (Wang et al., 2015; Connolly et al., 2019; Mudryk et al., 2020b; Khani et al., 2022). Interestingly, anomalously cold periods and large snowfalls in recent winters have been experienced in North America, Asia, and Europe (Kug et al., 2015), leading to increasing snow cover extent for some areas (Dahe et al., 2006; Roessler and Dietz, 2022; Young, 2023).

Climate models are periodically updated and reported by the Integovernmental Panel on Climate Change (IPCC) (IPCC, 2023). Mudryk et al. (2020) provided a more accurate analysis of Northern Hemisphere snow cover extent using the CMIP6 (World Climate Research Programme Coupled Model Intercomparison Project Phase 6) multi-model ensemble rather than CMIP5. Several studies have also been carried out to develop statistical models aimed at explaining the impact of climate changes on Arctic sea ice extent (Drobot, 2003; Drobot et al., 2006; Lindsay et al., 2008). However, there are only a few papers on forecasting ice sea and snow cover extent (e.g., Smith et al., 2016; Castle and Hendry, 2020; Essery et al., 2020; Blazsek et al., 2024).

All the above studies use standard regression models based on the assumption that the disturbances are well behaved, i.e. that they do not exhibit long-range dependence. In fact, as our analysis below will show, this is an incorrect assumption to make, and once allowance is made for the possibility of long memory no evidence can be found of a significant time trend. This suggests that the previous evidence might have been driven by an incorrect model specification.

3. Methodology

The most common approach to modelling trends in time series is to estimate a linear regression of the following form:

$$y(t) = \alpha + \beta t + x(t), \quad t = 1, 2, \ldots, \tag{1}$$

where α and β are unknown parameters, respectively a constant and the coefficient on the linear time trend, and x(t) is assumed to be well behaved, namely to satisfy some standard properties such as covariance stationarity and integration of order 0 (I(0)).

To be more specific, a covariance stationary process {x(t), t = 0, ±1, …} with mean μ is said to exhibit short memory or be I(0) if the sum of its autocovariances, defined as $\gamma(u) = E[(x_t - \mu)(x_{t+u} - \mu)]$, is finite, i.e.

$$\sum_{u=-\infty}^{u=\infty} | \gamma(u) | < \infty. \tag{2}$$

An alternative definition is based on the frequency domain – in this case a process is said to exhibit short memory if the spectral density function, f(λ) (which is the Fourier transform of the autocovariances) is positive and bounded at all frequencies in the interval [0, π), i.e.,

$$0 < f(\lambda) < \infty. \tag{3}$$

Examples of short-memory processes include the white noise one and the (stationary) AutoRegressive Moving Average (ARMA) models.

In the context of the model given by Equation (1), one can conclude that there is a significant time trend in the series of interest (the ice/snow cover extent in our case) if the following null hypothesis can be rejected:

$$H_o: \quad \beta = 0, \tag{4}$$

in favour of the one-sided alternative:

$$H_o: \quad \beta < 0, \tag{5}$$

An important issue here is the choice of the appropriate estimator for β in (1). For instance, if x(t) is a random variable independently drawn from a Gaussian distribution with mean zero and a constant variance, Ordinary Least Squares (OLS) will yield unbiased estimates of *β*, and statistical inference can be made using standard *F* and *t* statistics (see, e.g., Hamilton (1994). However, in the presence of weak autocorrelation, OLS or Generalized Least Squares (GLS) estimates are only valid if *x(t)* exhibits short memory. In other words, the error term x(t) must be integrated of order 0 ($x(t) \approx I(0)$), a condition that is not satisfied by many climatological series. In fact, many earth-related series appear to exhibit long memory, namely the infinite sum of their autocovariances is infinite, i.e.

$$\sum_{u=-\infty}^{u=\infty} |\, \gamma(u) \,| \;=\; \infty. \tag{6}$$

or, alternatively, in the frequency domain, their spectral density function (f(λ)) has at least one pole or singularity in the spectrum, i.e.,

$$f(\lambda) \to \infty, \quad as\ \lambda \ \to 0. \tag{7}$$

Examples of climatological series with long-memory properties can be found in several papers such as Stephenson et al., (2000), Gil-Alana (2005, 2008a, 2017), Vyushin and Kushner (2009), Zhu et al. (2010), Lennartz and Bunde (2009), Rea et al. (2011), Franzke (2011), Yuan et al. (2014a,b), and more recently, Yuan et al. (2022), Zhu et al. (2023), and Gil-Alana and Carmona (2023).

Within the category of long-memory models, a very simple one widely used by researchers is based on fractional integration, or I(d) with d > 0, with d-differencing being required to make the series of interest stationary I(0). Whilst the early literature only allowed for integer values of the parameter d (0 for stationary series or 1 for non-stationary ones), Granger (1980, 1981), Granger and Joyeux (1980), Hosking (1981) subsequently analysed the fractional case where d can be any real value.

Specifically, a process is said to be I(d) if it can be represented as:

$$(1-B)^d x(t) \;=\; u(t), \qquad t = 1,2,\ldots, \tag{8}$$

where B is a backshift function (i.e., Bx(t) = x(t-1)) and u(t) is an I(0) process defined as above. In this context, long memory occurs if d is positive, with the spectral density function of x(t) going to infinity as the frequency λ approaches zero:

$$f(\lambda) \rightarrow \infty, \quad as\ \lambda\ \rightarrow 0. \tag{9}$$

Note that the polynomial on the left-hand side of (8) can be expressed in terms of a Binomial expansion such that, for any real value d,

$$(1-B)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j B^j = 1 - dB + \frac{d(d-1)}{2} B^2 - \cdots \tag{10}$$

and thus equation (8) can be expressed as

$$x(t) \;=\; d\, x(t-1) \;-\; \frac{d(d-1)}{2}\, x(t-2) \;+\; \ldots \;+\; u(t). \tag{11}$$

In this context, if d is a non-integer value, x(t) will be a function of all its past history: the higher the value of d is, the higher will be the degree of dependence between observations. Moreover, if d is positive, the series exhibits long memory as explained above, and mean reversion takes place as long as d is smaller than 1, while stationarity holds if d < 0.5.

In the empirical application carried out in the following section, we consider the following model,

$$y(t) = \alpha + \ \beta\, t \ + \ x(t), \quad (1-B)^d x(t) = u(t), \quad u(t) = \rho u(t-12) + \ \varepsilon(t)\ , \tag{12}$$

where α and β are unknown parameters to be estimated, t is a time trend, B is the backshift operator; d refers to the number of differences required to make u(t) a I(0) process (a monthly AR process in the case of the monthly series under investigation in our case); finally, ε(t) is assumed to be a white noise process. We then carry out the Lagrange Multiplier (LM) test proposed in Robinson (1994) and widely used for long-memory and fractionally integrated processes (e.g., Gil-Alana and Robinson, 1997).

4. Data and Empirical Results

The data used for the analysis are (i) the monthly ice extent series produced by the National Snow and Ice Data Center (NSIDC) for the Globe, the Northern Hemisphere, and the Southern Hemisphere, from January 1979 to April 2024, and (ii) the monthly snow cover extent series constructed by the Rutgers University Global Snow Laboratory (GSL) for North America + Greenland, the Northern Hemisphere, and Europe + Asia, from January 1967 to April 2024[2]. These series are the result of a collaborative effort between The National Centers for Environmental Information (NCEI) - National Oceanic and Atmospheric Administration (NOAA) and the Rutgers University Global Snow Lab (GSL). All anomalies are relative to the 1991–2020 average. [https://www.ncei.noaa.gov/access/monitoring/snow-and-ice-extent/sea-ice/G/6]

Figure 1 contains plots of the time series. Visual inspection suggests the existence of a downward trend in most sea ice extent series (though this is less clear in the case of the southern hemisphere), whilst the same is not apparent in the case of the snow cover extent series.

Tables 1 – 3 report the estimates of the intercept, the time trend and the seasonal AR coefficient in the model given by Equation (12) under the assumption that d = 0, i.e., imposing that x(t) = u(t) and thus follows a short-memory process. More specifically, Table 1 reports the full sample estimates, i.e., from 1971m1 to 2024m12 for the sea ice cover extent and from 1972m1 to 2024 for the snow cover extent, whilst Tables 2 and 3 focus on the most recent periods, namely from 2000m1 and 2015m1 respectively.

INSERT TABLES 1 – 3 ABOUT HERE

[2] No snow extent data are available for the Southern Hemisphere from NOAA (2024).

Over the full sample the time trend coefficient β is negative and significantly different from 0 in all cases with the exception of the sea ice cover extent series for the southern hemisphere (Table 1). By contrast, when considering the period from 2000m1 (Table 2), a negative and significant trend is estimated for all three sea ice cover extent series as well as the Eurasian snow cover extent one; such a trend is also found over the period starting in 2015m1 (Table 3) for four out of the six series, namely sea ice cover extent in the Northern and Southern Hemispheres as well as snow cover extent in Europe + Asia and in the Northern Hemisphere. However, these results might be biased since they have been obtained by assuming short memory in the regression errors in (12) which, as previously argued, is questionable. Therefore we performed a short memory / long memory test, which is based on the modified Lo's (1991) statistic; the results strongly reject the null hypothesis of short memory in favour of long memory.

Therefore, in what follows we relax the short-memory assumption and estimate the differencing parameter d from the data along with the other parameters of interest. These results are displayed in Tables 4 - 6.

INSERT TABLES 4 – 6 ABOUT HERE

In particular, Table 4 shows that over the full sample the differencing parameter d is positive and significant in all cases; this implies the presence of long memory in the series, and thus invalidates the previous results reported in Table 1. The estimated value of this parameter is much higher for the sea ice cover extent than for the snow cover extent; more precisely, in the case of the former series the estimates of d are 0.85 for the globe, and 0.81 and 0.75 for the northern and southern hemispheres respectively, whilst in the case of the snow cover extent the estimated values are 0.41, 0.39 and 0.37 for the Europe +

Asia, North America + Greenland and northern hemisphere series respectively. Most interestingly, the time trend coefficients, though still negative, are now not significantly different from zero for any of the six series examined. As before, seasonality seems to be an important feature of the data.

Table 5 reports the results for the period starting in 2000m1. These estimates of d are rather similar to those in the previous case, with values ranging from 0.74 to 0.84 for the sea ice cover extent, and from 0.26 to 0.37 for the snow cover extent, but once again none of the β coefficients appear to be statistically significant. Finally, Table 6 shows the estimates for the period starting in 2015m1; it can be seen that the estimates of d are in a similar range, namely between 0.71 (northern hemisphere) and 0.86 (southern hemisphere) in the case of the sea ice cover extent, and between 0.24 (North America + Greenland) and 0.40 (northern hemisphere) in the case of the snow cover extent. Once more, all the β coefficients are insignificant, which again implies that the series being investigated do not exhibit a trend.

5. Conclusions

Global warming is one of the main contemporary issues for the planet Earth. One of its effects is thought to be a decline in sea ice and snow cover extent, which are crucial variables for the global climate system. The available evidence indeed seems to suggest that rising temperatures are leading to a marked and sustained fall in both those series (see, e.g., Pavlova, 2014; Mudryk, 2020a), this being an additional factor driving climate change.

The present study revisits this evidence by analysing monthly data on sea ice cover extent from January 1979 to April 2024 and on snow cover extent from January 1967 to April 2024 for various regions of the world by means of fractional integration methods.

The results can be summarised as follows. A statistically significant time trend is found when the errors in the regression model are incorrectly assumed to exhibit short memory. However, this evidence vanishes when the errors are allowed instead to follow an I(d) process with d different from zero and positive, and thus to be characterised by long memory (a well-known property of most climatological series)– in this case, the time trend becomes insignificant for all series. This implies that previous findings suggesting an irreversible decline in both sea ice and snow cover extent might be misleading because of an incorrect model specification. However, since human activities have unequivocally caused global warming (principally through emissions of greenhouse gases), with global surface temperature reaching 1.1°C above 1850–1900 in 2011–2020 (see, e.g. IPCC, 2023), the case for climate change policies remains overwhelming.

Future work could investigate additional issues including (i) the possible presence of structural breaks in the data, by following the Bai and Perron (2003) approach or alternatively carrying out the tests proposed by Gil-Alana (2008b) and Hassler and Meller (2014), both specifically designed for the case of fractional integration, (ii) non-linearities, applying methods based on Chebyshev's polynomials (Cuestas and Gil-Alana, 2016), Fourier transform functions (Gil-Alana and Yaya, 2021; Caporale et al., 2022) or neural networks (Yaya et al., 2021), again in the context of fractional integration, and (iii) cyclical patterns by estimating appropriate short- and long-memory models incorporating them (Caporale and Gil-Alana, 2024).

References

Bai, J., and Perron, P. 2003. "Computation and analysis of multiple structural change models," Journal of Applied Econometrics, 18(1), 1-22.

Brown, R.; Derksen, C.; Wang, L. (2010), A multi-data set analysis of variability and change in Arctic spring snow cover extent, 1967–2008. Journal of Geophysical Resarch Atmosphere 115, D16.

Brown, R.; Robinson, D. (2011), Northern Hemisphere spring snow cover variability and change over 1922–2010 including an assessment of uncertainty. Cryosphere 5, 219–229.

Blazsek, S. Escribano, A. and Kristof, E. (2024) Global, Arctic, and Antarctic sea ice volume predictions using score-driven threshold climate models. Energy Economics 134 (2024) 107591.

Brutel-Vuilmet, C.; Ménégoz, M.; and Krinner, G. (2013) An analysis of present and future seasonal Northern Hemisphere land snow cover simulated by CMIP5 coupled climate models. The Cryosphere, 7, 67–80. doi:10.5194/tc-7-67-2013.

Callaghan, T.V. ; Johansson, M.; Brown, R. D.; Ya, P. *et al*. (2011) Multiple Effects of Changes in Arctic Snow Cover. AMBIO 40:32-45. DOI 10.1007/s13280-011-0213-x.

Caporale, G.M. and L.A. Gil-Alana (2024). A long-memory model for multiple cycles with an application to the US stock market", Mathematics, forthcoming.

Caporale, G.M., Gil-Alana, L. and O.S. Yaya (2022), "Modelling persistence and non-linearities in the US Treasury 10-year bond yields", Economics Bulletin, 42, 3.

Castle, J., Hendry, D., 2020. Climate econometrics: An overview. Foundations and Trends® in Econometrics. 10 (3–4), 145–322. http://dx.doi.org/10.1561/0800000037.

Cavalieri, D.J. and Parkinson, C.L. (2012) Arctic sea ice variability and trends, 1979–2010. Cryosphere, 6, 881–889 (doi: 10.5194/tc-6-881-2012).

Chahine, M.T. (1992), The hydrological cycle and its influence on climate. Nature 359, 373–380.

Comiso J.C. and Nishio F. (2008), Trends in the sea ice cover using enhanced and compatible AMSR-E, SSM/I, and SMMR data. Journal of Geophysical Research, 113, C02S07 (doi:10.1029/2007JC004257).

Connolly, R., M. Connolly, W. Soon, D. Legates, R.G. Cianco and V.M. Velasco-Herrera (2019) Northern Hemisphere Snow-Cover Trends (1967–2018): A Comparison between Climate Models and Observations", Geosciences 9, 135; doi:10.3390/geosciences9030135

Cuestas J.C. and L.A. Gil-Alana (2016). "A Non-Linear Approach with Long Range Dependence Based on Chebyshev Polynomials", Studies in Nonlinear Dynamics and Econometrics, 23, 445–468.

Dahe, Q.; Shiyin, L.; and Peiji, L. (2006) Snow Cover Distribution, Variability, and Response to Climate Change in Western China. Journal of Climate 19, 1820-1833.

Déry, S. J. and Brown, R. D. (2007) Recent Northern Hemisphere snow cover extent trends and implications for the snow-albedo feedback. *Geophysical Research Letters* 34, L22504; doi:10.1029/2007GL031474.

Drobot, S. (2003). Long-range statistical forecasting of ice severity in the Beaufort–Chukchi Sea. Weather Forecast. 2003, 18, 1161–1176.

Drobot, S.D.; Maslanik, J.A.; Fowler, C. (2006) A long-range forecast of Arctic summer sea-ice minimum extent. Geophysical Research Letters *33*, L10501.

Essery, R.; Kim, H.; Wang, L.; Bartlett, P.; Boone, A.; Brutel-Vuilmet, C.; Burke, E.; Cuntz, M.; Decharme, B.; Dutra, E.; Fang, X.; Gusev, Y.; Hagemann, S.; Haverd, V.; Kontu, A.; Krinner, G.; Lafaysse, M.; Lejeune, Y.; Marke, T.; Marks, D.; Marty, C.; Menard, C.B.; Nasonova, O.; Nitta, T., Pomeroy, J.; Schädler, G.; Semenov, V.; Smirnova, T.; Swenson, S.; Turkov, D.; Wever, N., and Yuan, H. (2020) Snow cover duration trends observed at sites and predicted by multiple models. The Cryosphere, 14, 4687–4698. https://doi.org/10.5194/tc-14-4687-2020

Estilow T.W., Young A.H. and Robinson D.A. (2015) A long-term Northern Hemisphere snow cover extent data record for climate studies and monitoring, *Earth System Science Data* 7, 137–42.

Franzke, C. (2010), Long-range dependence and climate noise characteristics of Antarctic temperature data, Journal of Climate 23(22), 6074-6121.

Frei, A.; Tedesco, M.; Lee, S.; Foster, J.; Hall, D.K.; Kelly, R.; Robinson, D.A. (2012), A review of global satellite-derived snow products. Advanced. Space Research 50, 1007–1029],

Gil-Alana, L.A. (2005). Statistical model for the temperatures in the Northern hemisphere using fractional integration techniques, Journal of Climate, 18(24), 5537–5369. https://doi.org/10.1175/JCLI3543.1

Gil-Alana, L.A. (2008a). Warming Break Trends and Fractional Integration in the Northern, Southern, and Global Temperature Anomaly Series, Journal of Atmospheric and Oceanic Technology 25 (4), 570-578 https://doi.org/10.1175/2007JTECHA1025.1

Gil-Alana, L.A. (2008b). Fractional integration and structural breaks at unknown periods of time. Journal of Time Series Analysis, 29, 163-185.

Gil-Alana, L.A. and Carmona, N. (2023), Temperature Anomalies in the Northern and Southern Hemispheres. Evidence of Persistence and Trends, Research Square, https://doi.org/10.21203/rs.3.rs-3256176/v1

Gil-Alana, L.A., Robinson, P.M. (1997). Testing of unit roots and other nonstationary hypotheses, Journal of Econometrics 80, 2, 241-268.

Gil-Alana, L.A. and O. Yaya (2021). Testing fractional unit roots with non-linear smooth break approximations using Fourier functions. Journal of Applied Statistics 48 (13-15), 2542-2559.

Gouttevin, I., Menegoz, M., Domine, F., Krinner, G., Koven, C., Ciais, P., Tarnocai, C., and Boike, J. (2012) How the insulating properties of snow affect soil carbon distribution in the continental pan-Arctic area, Journal of Geophysical Research 117, G02020, doi:10.5194/tc6-407-2012.

Granger, C.W.J. (1980). Long memory relationships and the aggregation of dynamic models, Journal of Econometrics 14, 2, 227-238.

Granger, C.W.J. (1981). Some properties of time series data and their use in econometric model specification, Journal of Econometrics 16, 121-130.

Granger, C.W.J., Joyeux, R. (1980). An introduction to long-memory time series models and fractional differencing. Journal of Time Series Analysis. 1, 15–29.

Groisman, P.Y., T.R. Karl, and R.W. Knight (1994a), Observed impact of snow cover on the heat balance and the rise of continental spring temperatures, Science, 263, 198 – 200

Groisman, P.Y., T.R. Karl, R.W. Knight, and G.L. Stenchikov (1994b), Changes of snow cover, temperature and radiative heat balance over the Northern Hemisphere, Journal of Climate 7, 1633 – 1656].

Hamilton, J. D., 1994: *Time Series Analysis*. Princeton University Press, 820 pp.

Hassler, U., and B. Meller, 2014. "Detecting multiple breaks in long memory. The case of US inflation," Empirical Economics 46, 2, 653-680.

Hosking, J.R. (1981). Fractional differencing. Biometrika, 68, 165-176.

IPCC, 2023: Sections. In: Climate Change 2023: Synthesis Report. Contribution of Working Groups I, II and III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change [Core Writing Team, H. Lee and J. Romero (eds.)]. IPCC, Geneva, Switzerland, pp. 35-115, doi: 10.59327/IPCC/AR6-9789291691647.

Mohammadzadeh Khani, H.; Kinnard, C.; Lévesque, E. (2022) Historical Trends and Projections of Snow Cover over the High Arctic: A Review. Water 14, 587. https://doi.org/10.3390/w14040587

Kug, J.S.; Jeong, J.H.; Jang, Y.S.; Kim, B.M.; Folland, C.K.; Min, S.K.; Son, S.W. (2015) Two distinct influences of Arctic warming on cold winters over North America and East Asia. National Geoscience 8, 759–762.

Kukla, G.J. and Kukla, H.J. (1974), Increased surface albedo in the Northern Hemisphere. Science 183, 709–714.

Lawrence, D. M. and Slater, A. G. (2010) The contribution of snow condition trends to future ground climate, Climate Dynamics 34, 969–981.

Lennartz, S. and A. Bunde (2009), Trend evaluation in records with long-term memory: Application to global warming, Geophysical Research Letters 36(16)

Li, Q.; Ma, M.; Wu, X. and Yang, H. (2018), Snow cover and vegetation-induced decrease in global albedo from 2002 to 2016. Journal of Geophysical Research Atmosphere 123, 124–138].

Lindsay, R.W.; Zhang, J.; Schweiger, A.J.; Steele, M.A. (2008). Seasonal predictions of ice extent in the Arctic Ocean. Journal of Geophysical Research 113, C02023.

Marty, C., Tilg, A.-M., and Jonas, T. (2017). Recent evidence of large-scale receding snow water equivalents in the European Alps, J. Hydrometeorol., 18, 4, 1021–1031, https://doi.org/10.1175/JHM-D16-0188.1.

Mudryk, L., Santolaria-Otín, M., Krinner, G., Ménégoz, M., Derksen, C., Brutel-Vuilmet, C., Brady, M., and Essery, R. (2020a) Historical Northern Hemisphere snow cover trends and projected changes in the CMIP6 multi-model ensemble, The Cryosphere, 14, 2495–2514. https://doi.org/10.5194/tc-14-2495-2020.

Mudryk L.; Chereque, A.E.; Brown, R.; Derksen, C.; Luojus, K. and Decharme, B. (2020b) Arctic report card 2020: terrestrial snow cover. NOAA. https://arctic.noaa.gov/report-card/report-card-2020/terrestrial-snow-cover-3/

Mudryk, L.; Chereque, A.E.; Brown, R.; Derksen, C.; Luojus, K. and Decharme, B. (2021), Arctic report card 2021: terrestrial snow cover, NOAA. https://arctic.noaa.gov/Report-Card/Report-Card-2021.

Najafi, M.R.; Zwiers, F.W. and Gillett, N.P. (2016) Attribution of the spring snow cover extent decline in the Northern Hemisphere, Eurasia and North America to anthropogenic influence. Climate Change 136, 571–586]

NOAA, 2024: National Oceanic and Atmospheric Administration (NOAA). https://www.ncei.noaa.gov/access/monitoring/monthly-report/global-snow/202401.

NOAA/Rutgers Global Snow Labhttps://globalcryospherewatch.org/state_of_cryo/snow/

Pavlova, O.; Pavlov, V.; Gerland, S. (2014) The impact of winds and sea surface temperatures on the Barents Sea ice extent: A statistical approach. Journal of Marine Systems 130, 248–255.

Parkinson C.L. (2014) Global sea ice coverage from satellite data: annual cycle and 35-yr trends. International Journal of Climatology 27, 9377–9382 (doi: 10.1175/JCLI-D-14-00605.1)].

Parkinson, C.L., Cavalieri DJ, Gloersen P, Zwally HJ and Comiso JC (1999) Arctic sea ice extents, areas, and trends, 1978–1996. J. Geophys. Res., 104(C9), 20,837–20,856].

Peng, G. and Meier, W.N. (2017) Temporal and regional variability of Arctic sea-ice coverage from satellite data. Annals of Glaciology 59(76pt2): 191–200 (doi: 10.1017/aog.2017.32)

Rea, W., M. Reale and J. Brown (2011), Long memory in temperature reconstructions, Climate Change 107(3-4): 247-265.
https://doi.org/10.1007/s10584-011-0068-y

Robinson, P.M. (1994). Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association 89, 1420-1437.

Roessler, S. and Dietz, A.J. (2022), Development of Global Snow Cover—Trends from 23 Years of Global SnowPack. Earth 4, 1–22.

Smith, G., Roy, F., Reszka, M., Colan, D., He, Z., Deacu, D., Belanger, J.-M., Skachko, S., Liu, Y., Dupont, F., Lemieux, J.-F., Beaudoin, C., Tranchant, B., Drevillon, M., Garric, G., Testut, C.-E., Lellouche, J.-M., Pellerin, P., Ritchie, H., Lu, Y., Davidson, F., Buehner, M., Caya, A., Lajoie, M., (2016) Sea ice forecast verification in the Canadian global ice ocean prediction system. Q. J. R. Meteorol. Soc. 142 (B), 659–671. http://dx.doi.org/10.1002/qj.2555.

Stephenson DB, Pavan V and Bojariu R (2000) Is the North Atlantic oscillation a random walk?, International Journal of Climatology 20,1–18

Sturm, M., and Massom, R. A. (2009). “Snow and sea ice,” in Sea ice, eds D. N. Thomas and G. S. Dieckmann.

Vyushin, D.I. and Kushner, P.J. (2009). Power-Law and Long-Memory Characteristics of the Atmospheric General Circulation, Journal of Climate 22 (11), 2890–2904, https://doi.org/10.1175/2008JCLI2528.1

Wang, T.; Peng, S.; Ottlé, C. and Ciais, P. (2015) Spring snow cover deficit controlled by intraseasonal variability of the surface energy fluxes. Environmental Research Letter 10, 024018

Yaya, O.S., Ogbonna, A.E., Furuoka, F., & Gil-Alana, L.A. (2021). A new unit root test for unemployment hysteresis based on the autoregressive neural network. Oxford Bulletin of Economics and Statistics, 83(4), 960-981.

Young, S.S. (2023) Global and Regional Snow Cover Decline: 2000–2022. Climate 11, 162. https://doi.org/10.3390/cli11080162].

Yuan, N., Fu, Z. and Liu, S. (2014a). Long-term memory in climate variability: A new look based on fractional integral techniques, Journal of Geophysical Research Atmospheres 118(23), 12962-12969, https://doi.org/10.1002/2013JD020776

Yuan, N., Fu, Z. and Liu, S. (2014b). Extracting climate memory using Fractional Integrated Statistical Model: A new perspective on climate prediction. Scientific Report 4 (1). https://doi.org/10.1038/srep06577

Yuan, N., Franzke, C.L.E., Xiong, F. *et al.* (2022). The impact of long-term memory on the climate response to greenhouse gas emissions. NPJ Climate Atmospheric Science 5(70), https://doi.org/10.1038/s41612-022-00298-8.

Zhang, T. (2005). Influence of the seasonal snow cover on the ground thermal regime: An overview, Reviews of Geophysics 43, RG4002, doi:10.1029/2004RG000157.

Zhu, X, K. Fraedich, Z. Liu and R. Blender (2010), A Demonstration of Long-Term Memory and Climate Predictability, Journal of Climate 23(18).

Zhu, J., Xie, A., Qin, X., Xu, B. and Wang, Y. (2023). Assessment of Antarctic Amplification Based on a Reconstruction of Near-Surface Air Temperature. Atmosphere 2023, 14, 218. https://doi.org/10.3390/atmos14020218

Figure 1: Time series plots

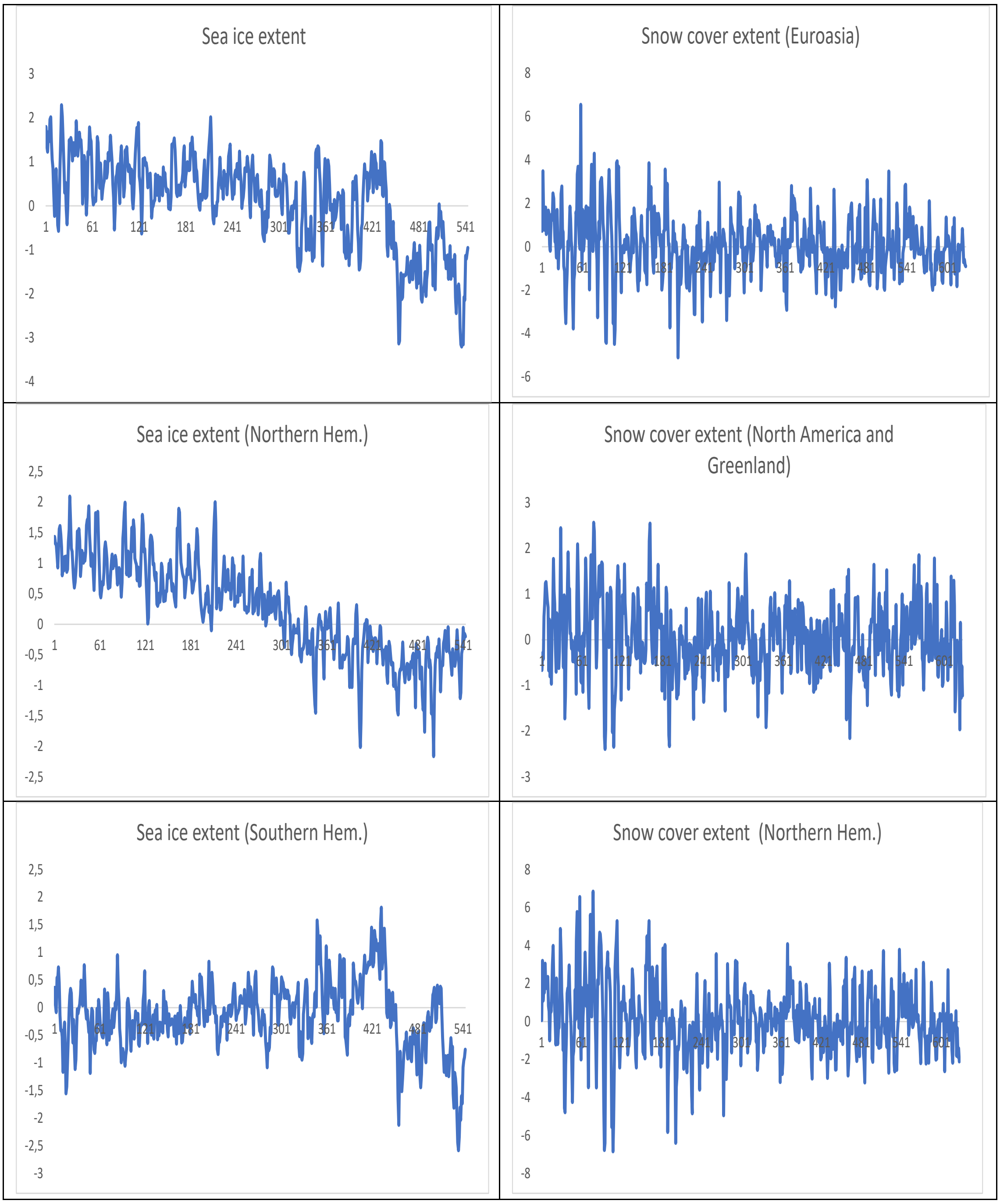

Table 1: Estimated coefficients in Equation (12) and d = 0 using the full sample

Series	Intercept	Time trend	Seasonal AR
Sea Ice Extent (1979m1 – 2024m12)			
Sea Ice Extent (Global)	1.3363 (21.29)*	-0.00451 (-22.64)*	0.329
Sea Ice Extent (Northern. H.)	1.4158 (40.13)*	-0.00428 (-38.24)*	0.448
Sea Ice Extent (Southern H.)	-0.0780 (-1.47)	-0.00022 (-1.35)	0.407
Snow Coverage Extent (1972qm1 – 2024m12)			
Europe + Asia	0.4090 (3.74)*	-0.00107 (-3.55)*	0.389
North + Greenland	0.1648 (2.62)*	-0.00042 (-2.43)*	0.269
Northern Hem.	0.5723 (3.89)*	-0.00149 (-3.68)*	0.472

*: Significance of the intercept and time trend coefficients at the 5% level.

Table 2: Estimated coefficients in Equation (12) with d = 0 using data starting in 2000m1

Series	Intercept	Time trend	Seasonal AR
Sea Ice Extent (1979m1 – 2024m12)			
Sea Ice Extent (Global)	0.7217 (7.92)*	-0.00747 (-14.25)*	0.321
Sea Ice Extent (Northern. H.)	0.3162 (6.70)*	-0.00041 (-15.13)*	0.444
Sea Ice Extent (Southern H.)	0.4071 (5.07)*	-0.00335 (-7.26)*	0.433
Snow Coverage Extent (1972qm1 – 2024m12)			
Europe + Asia	0.1474 (1.18)	-0.00126 (-1.76)*	0.190
North + Greenland	0.0100 (0.12)	-0.00009 (-0.04)	0.054
Northern Hem.	0.1564 (0.99)	-0.00135 (-1.49)	0.286

*: Significance of the intercept and time trend coefficients at the 5% level.

Table 3: Estimated coefficients in Equation (12) with d = 0 using data starting in 2015m1

Series	Intercept	Time trend	Seasonal AR
Sea Ice Extent (1979m1 – 2024m12)			
Sea Ice Extent (Global)	-0.2396 (-1.57)	-0.01484 (-6.80)*	0.245
Sea Ice Extent (Northern. H.)	-0.6752 (-9.81)*	-0.00050 (-0.51)	0.418
Sea Ice Extent (Southern H.)	0.4372 (3.43)*	-0.01535 (-8.41)*	0.279
Snow Coverage Extent (1972qm1 – 2024m12)			
Europe + Asia	0.2408 (1.36)	-0.00592 (-2.33)*	0.403
North + Greenland	0.1106 (0.83)	-0.00195 (-1.02)	0.114
Northern Hem.	0.3494 (1.42)	-0.00785 (-2.23)*	0.462

*: Significance of the intercept and time trend coefficients at the 5% level.

Table 4: Estimated coefficients in Equation (12) using the full sample

Series	d (95% band)	Intercept	Time trend	Seasonal
Sea Ice Extent (1979m1 – 2024m12)				
Ice (Global)	0.85 (0.74, 0.97)	1.7146 (3.99)*	-0.0052 (-0.67)	0.127
Ice (Northern. H.)	0.81 (0.68, 0.95)	1.3975 (5.08)*	-0.0034 (-0.85)	0.351
Ice (Southern H.)	0.75 (0.67, 0.85)	0.2564 (0.82)	-0.0018 (-0.54)	0.070
Snow Coverage Extent (1972qm1 – 2024m12)				
Europe + Asia	0.41 (0.32, 0.52)	0.8264 (1.40)	-0.0018 (-1.07)	0.315
North + Greenland	0.37 (0.29, 0.47)	1.1512 (0.50)	-0.0049 (-0.57)	0.191
Northern Hem.	0.39 (0.30, 0.50)	0.9287 (1.27)	-0.0023 (-1.08)	0.377

*: Significance of the intercept and time trend coefficients at the 5% level.

Table 5: Estimated coefficients in Equation (12) using data starting in 2000m1

Series	d (95% band)	Intercept	Time trend	Seasonal
Sea Ice Extent (1979m1 – 2024m12)				
Ice (Global)	0.84 (0.72, 1.00)	0.7959 (1.79)*	-0.0070 (-0.62)	0.118
Ice (Northern. H.)	0.74 (0.59, 0.92)	0.7318 (2.76)	-0.0040 (-0.95)	0.317
Ice (Southern H.)	0.80 (0.69, 0.95)	0.0317 (0.09)	-0.0033 (-0.47)	0.099
Snow Coverage Extent (1972qm1 – 2024m12)				
Europe + Asia	0.36 (0.25, 0.50)	0.9505 (1.45)	-0.0044 (-1.15)	0.413
North + Greenland	0.27 (0.18, 0.39)	0.2604 (0.87)	-0.0012 (-0.81)	0.306
Northern Hem.	0.34 (0.22, 0.48)	0.2136 (0.39)	-0.0021 (-0.68)	0.252

*: Significance of the intercept and time trend coefficients at the 5% level.

Table 6: Estimated coefficients in Equation (12) using data starting in 2015m1

Series	d (95% band)	Intercept	Time trend	Seasonal
Sea Ice Extent (1979m1 – 2024m12)				
Ice (Global)	0.79 (0.66, 0.98)	0.9232 (2.08)*	-0.0190 (-1.12)	0.025
Ice (Northern. H.)	0.71 (0.54, 0.95)	-0.3078 (-1.17)	-0.0004 (-0.06)	0.480
Ice (Southern H.)	0.86 (0.71, 1.10)	1.2236 (3.46)*	-0.0181 (-1.02)	0.036
Snow Coverage Extent (1972qm1 – 2024m12)				
Europe + Asia	0.38 (0.23, 0.58)	-0.0080 (-1.60)	-0.0036 (-0.50)	0.460
North + Greenland	0.24 (0.11, 0.42)	0.0717 (0.26)	-0.0023 (-0.62)	0.070
Northern Hem.	0.40 (0.25, 0.62)	-0.0459 (-0.06)	-0.0058 (-0.54)	0.493

*: Significance of the intercept and time trend coefficients at the 5% level.