An iterative LMI approach to H_∞ networked control with random communication delays

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Abstract: In this paper, an iterative LMI approach is proposed to deal with the control problem for Networked Control Systems (NCSs) with random communication delays. Both random measurement and control delays are simultaneously considered due to limited communication capacity. The NCSs with both measurement and control delays are modelled as a stochastic parameter system which contains two independent Bernoulli distributed white sequence. A dynamic output controller is designed to exponentially stabilise the networked system in the sense of mean square, and also achieve the prescribed H_∞ disturbance attenuation level. An iterative algorithm is developed to compute the optimal H_∞ disturbance attenuation and the controller parameters. An illustrative example is provided to show the applicability of the proposed method.

Keywords: NCS; networked control system; random communication delay; stochastic parameter system; H_∞ control; iterative LMI approach.

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1 Introduction

Recent advances in network technology have led to more and more control systems which form the feedback control loop through a network. This kind control system is called Networked Control System (NCS). The network itself is a dynamic system and will induce the delays via network communication due to limited bandwidth. A successful NCS design should take the communication delays into account, since the delays are widely known to degrade the performance of the control system. Therefore, the control problem of networked system with delays has received increasing attention, see e.g., Goodwin et al. (2004), Hu and Zhu (2003), Krtolica et al. (1994), Matveev and Savkin (2001), Nilsson et al. (1998), Srinivasagupta et al. (2004), and Tipsuwan and Chow (2003), references therein.

Since network delays are usually random and time-varying, the existing *deterministic* time-delay control methods can not be directly used for the analysis and design of the NCS Yang et al. (2006, 2007). Recently, there have been significant research efforts on the control problems for networked systems with random delays. The random network delays have been modelled by using various formulations based on probability and the characteristics of sources and destinations. For example, in Nilsson et al. (1998) the time delays are varying in a random fashion and have statistically mutually independent transfer-to-transfer probability distribution. In Srinivasagupta et al. (2004) the random communication delays have been considered as white in nature with known probability distributions. In Kolmanovsky and Maizenberg (2001) the delay value is treated as an unknown variable but with known statistical properties, modelled by a Markov process with a finite number of states. The probabilistic delay averaging approach is employed to determine the optimal control in the form which is independent of the delay value. In Luck and Ray (1990), Ray (1994), Tsai and Ray (1997), and Yaz and Ray (1996), the system measurement mode with binary switching random delay was established, where the binary switching sequence was viewed as a Bernoulli distributed white sequence taking on values of 0 and 1. The estimation and control problems have been studied in their paper. The techniques to deal with random delay range from simple approaches to more sophisticated approaches.

In this paper, we will continue our previous research on the networked control with random communication delays (Yang et al., 2006), where a networked controller based on an observer is designed and the LMI problem with equality constraint has to be solved. An iterative LMI approach is proposed to solve the H_∞ networked control problem based on dynamic output controller. The NCSs with both random measurement and control delays are modelled as a stochastic parameter system with one-step delay which contains two independent Bernoulli distributed white sequence. A dynamic output controller is designed to exponentially stabilise the networked system in the sense of mean square and also achieve the prescribed H_∞ disturbance attenuation level. An iterative algorithm is developed to compute the optimal H_∞ disturbance attenuation and the controller parameters.

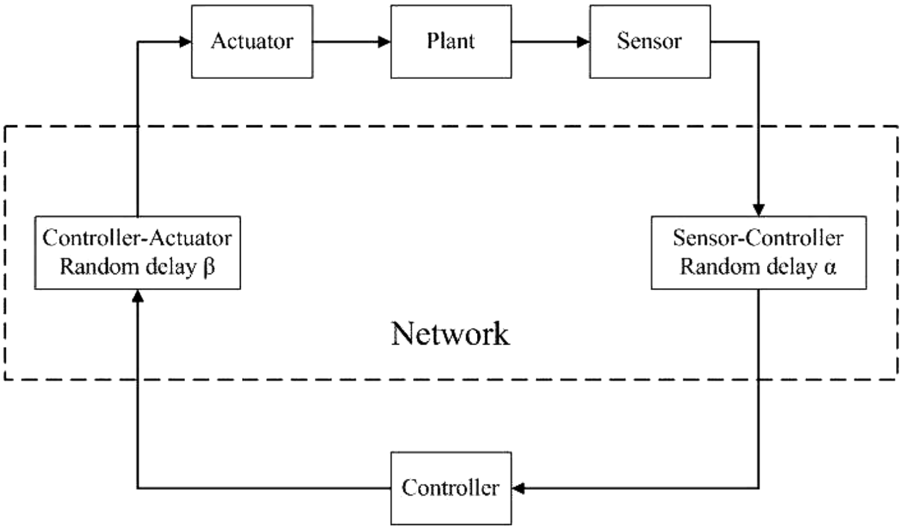
The rest of this paper is organised as follows. The control problem is formulated in Section 2 for NCSs with random communication delays. The stability with random delays is analysed and the H_∞ dynamic output controller is designed in Section 3, which achieve a desired H_∞ disturbance rejection. A simulation result is given in Section 4 to demonstrate the effectiveness of the proposed method. Conclusions are drawn in Section 5.

Notation. The notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. Prob$\{\cdot\}$ means the occurrence probability of the event '$\cdot$'. If A is a matrix, $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) means the largest (respectively, smallest) eigenvalue of A. $l_2[0, \infty)$ is the space of square integrable vectors, and $\mathbb{I}^+$ is the set of positive integer. In symmetric block matrices, '*' is used as an ellipsis for terms induced by symmetry.

2 Problem formulation and preliminaries

Consider the NCS with random communication delays shown in Figure 1.

Figure 1 The structure of a Networked Control System with random communication delays



The plant is assumed to be of the form

$$\begin{cases} x(k+1) = Ax(k) + B_1 w(k) + B_2 u_c(k), \\ z(k) = C_1 x(k) + D_{11} w(k) + D_{12} u_c(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u_c(k) \in \mathbb{R}^m$ is the control input, $z(k) \in \mathbb{R}^r$ is the controlled output, $w(k) \in \mathbb{R}^q$ is the disturbance input belonging to $l_2[0, \infty)$, A, B_1, B_2, C_1, D_{11} and D_{12} are known real matrices with appropriate dimensions. The measurement with random communication delays is described by

$$\begin{cases} y(k) = C_2 x(k) \\ y_c(k) = \alpha(k) y(k) + (1 - \alpha(k)) y(k-1), \end{cases} \tag{2}$$

where the stochastic variable $\alpha(k) \in \mathbb{R}$ is a Bernoulli distributed white sequence with

$$\text{Prob}\{\alpha(k) = 1\} = \mathbb{E}\{\alpha(k)\} := \bar{\alpha} \tag{3}$$

$$\text{Prob}\{\alpha(k) = 0\} = 1 - \mathbb{E}\{\alpha(k)\} := 1 - \bar{\alpha}, \tag{4}$$

and $y_c(k) \in \mathbb{R}^p$ is the measured output vector, $y(k) \in \mathbb{R}^p$ is the output vector, and C_2 is known real matrix with appropriate dimension.

Similar to the measurement channel, the control signals sent by the remote controller to the plant via the communication channel can be described by

$$u_c(k) = \beta(k) u(k) + (1 - \beta(k)) u(k-1), \tag{5}$$

where the stochastic variable $\beta(k) \in \mathbb{R}$, mutually independent of $\alpha(k)$, is also a Bernoulli distributed white sequence with

$$\text{Prob}\{\beta(k) = 1\} = \mathbb{E}\{\beta(k)\} := \bar{\beta} \tag{6}$$

$$\text{Prob}\{\beta(k) = 0\} = 1 - \mathbb{E}\{\beta(k)\} := 1 - \bar{\beta}. \tag{7}$$

In this paper, we consider the following dynamic controller for system (1):

$$\begin{cases} \hat{x}(k+1) = A_K \hat{x}(k) + B_K y_c(k), \\ u(k) = C_K \hat{x}(k), \end{cases} \tag{8}$$

where $\hat{x}(k)$ is the controller state, and A_K, B_K and C_K are the parameters to be determined.

From equations (1), (2), (5) and (8), the closed-loop system becomes:

$$\begin{cases} x_{cl}(k+1) = A_{cl} x_{cl}(k) + A_{dcl} x_{cl}(k-1) + B_{cl} w(k), \\ z(k) = C_{cl} x_{cl}(k) + C_{dcl} x_{cl}(k-1) + D_{cl} w(k), \end{cases} \tag{9}$$

where

$$x_{cl}(k) = \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix}, \quad A_{cl} = \begin{bmatrix} A & \beta(k) B_2 C_K \\ \alpha(k) B_K C_2 & A_K \end{bmatrix},$$

$$A_{dcl} = \begin{bmatrix} 0 & (1-\beta(k)) B_2 C_K \\ (1-\alpha(k)) B_K C_2 & 0 \end{bmatrix}, \quad B_{cl} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix},$$

$$C_{cl} = \begin{bmatrix} C_1 & \beta(k) D_{12} C_K \end{bmatrix}, \quad C_{dcl} = \begin{bmatrix} 0 & (1-\beta(k)) D_{12} C_K \end{bmatrix}, \quad D_{cl} = D_{11}.$$

In order to simplify the following analysis, we separate the stochastic parameters and deterministic parameters. To do this, we denote

$$A_{cl0} = \begin{bmatrix} A & \bar{\beta} B_2 C_K \\ \bar{\alpha} B_K C_2 & A_K \end{bmatrix}, \quad A_{cl1} = \begin{bmatrix} 0 & 0 \\ B_K C_2 & 0 \end{bmatrix}, \quad A_{cl2} = \begin{bmatrix} 0 & B_2 C_K \\ 0 & 0 \end{bmatrix}, \tag{10}$$

$$A_{dcl0} = \begin{bmatrix} 0 & (1-\bar{\beta}) B_2 C_K \\ (1-\bar{\alpha}) B_K C_2 & 0 \end{bmatrix}, \quad A_{dcl1} = \begin{bmatrix} 0 & 0 \\ -B_K C_2 & 0 \end{bmatrix},$$

$$A_{dcl2} = \begin{bmatrix} 0 & -B_2 C_K \\ 0 & 0 \end{bmatrix}, \tag{11}$$

$$C_{cl0} = \begin{bmatrix} C_1 & \bar{\beta} D_{12} C_K \end{bmatrix}, \quad C_{cl2} = \begin{bmatrix} 0 & D_{12} C_K \end{bmatrix}, \tag{12}$$

$$C_{dcl0} = \begin{bmatrix} 0 & (1-\bar{\beta}) D_{12} C_K \end{bmatrix}, \quad C_{dcl2} = \begin{bmatrix} 0 & -D_{12} C_K \end{bmatrix}, \tag{13}$$

then equation (9) is rewritten as

$$\begin{cases} x_{cl}(k+1) = A_{cl0} x_{cl}(k) + A_{dcl0} x_{cl}(k-1) \\ \qquad\qquad + (\alpha(k) - \bar{\alpha})(A_{cl1} x_{cl}(k) + A_{dcl1} x_{cl}(k-1)) \\ \qquad\qquad + (\beta(k) - \bar{\beta})(A_{cl2} x_{cl}(k) + A_{dcl2} x_{cl}(k-1)) + B_{cl} w(k), \\ z(k) = C_{cl0} x_{cl}(k) + C_{dcl0} x_{cl}(k-1) \\ \qquad\qquad + (\beta(k) - \bar{\beta})(C_{cl2} x_{cl}(k) + C_{dcl2} x_{cl}(k-1)) + D_{cl} w(k), \end{cases} \tag{14}$$

Since the closed-loop system (14) contains two stochastic quantities $\alpha(k)$ and $\beta(k)$, it is actually a stochastic parameter system and we need to introduce the notion of stochastic stability in the mean-square sense for the problem formulation.

Definition 1: The closed-loop system (14) is said to be *exponentially mean-square stable* if with $w(k) = 0$, there exist constants $\kappa > 0$ and $\tau \in (0, 1)$ such that

$$\mathbb{E}\{\|x_{cl}(k)\|^2\} \leq \kappa\tau^k \mathbb{E}\{\|x_{cl}(0)\|^2\}, \quad \text{for all } x_{cl}(0) \in \mathbb{R}^n, \quad k \in \mathbb{I}^+. \tag{15}$$

With this definition, our objective is to design the controller (8) for the system (1) such that, for both measurement and control packet communication delays (2) and (5), the closed-loop system (14) is exponentially mean-square stable, and the H_∞ performance constraint is satisfied. In other words, we aim to design a controller such that the closed-loop system satisfies the following requirements (Q1) and (Q2) simultaneously:

($Q1$) The closed-loop system (14) is exponentially mean-square stable.

($Q2$) Under the zero-initial condition, the controlled output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|z(k)\|^2\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|w(k)\|^2\}, \tag{16}$$

for all nonzero $w(k)$, where $\gamma > 0$ is a prescribed scalar.

3 Main results

In this section, we will first investigate the stability condition for the closed-loop system (14). The following lemma will be needed in our derivation.

Lemma 1: *Let $V(\eta(k))$ be a Lyapunov functional. If there exist real scalars $\lambda \geq 0$, $\mu > 0$, $\nu > 0$ and $0 < \psi < 1$ such that*

$$\mu\|\eta(k)\|^2 \leq V(\eta(k)) \leq \nu\|\eta(k)\|^2, \tag{17}$$

and

$$\mathbb{E}\{V(\eta(k+1)) \,|\, \eta(k)\} - V(\eta(k)) \leq \lambda - \psi V(\eta(k)). \tag{18}$$

then the sequence $\eta(k)$ satisfies

$$\mathbb{E}\{\|\eta(k)\|^2\} \leq \frac{\nu}{\mu}\|\eta(0)\|^2(1-\psi)^k + \frac{\lambda}{\mu\psi}. \tag{19}$$

Proof: The proof is similar to Tarn and Rasis (1976). □

The following theorem shows the closed-loop system (14) is exponentially stable in the mean-square sense.

Theorem 1: *Given the controller* (8)*. The closed-loop system* (14) *is exponentially mean-square stable if there exist positive definite matrices P and Q satisfying*

$$\begin{bmatrix} Q-P & * & * & * & * \\ 0 & -Q & * & * & * \\ PA_{cl0} & PA_{dcl0} & -P & * & * \\ aPA_{cl1} & aPA_{dcl1} & 0 & -aP & * \\ bPA_{cl2} & bPA_{dcl2} & 0 & 0 & -bP \end{bmatrix} < 0, \tag{20}$$

where

$$a = (1-\bar{\alpha})\bar{\alpha}, \quad b = (1-\bar{\beta})\bar{\beta}.$$

Proof: Define a Lyapunov functional

$$V(\eta_k) = \eta^T(k)\,\mathrm{diag}\{P, Q\}\eta(k), \tag{21}$$

where

$$\eta^T(k) = \begin{bmatrix} x_{cl}^T(k) & x_{cl}^T(k-1) \end{bmatrix}$$

and P and Q are positive definite matrices. It follows from (14) that

$$\begin{aligned}
&\mathbb{E}\{V(\eta(k+1))\,|\,V(\eta(k))\} - V(\eta(k)) \\
&\quad = \mathbb{E}\big\{x_{cl}^T(k+1)Px_{cl}(k+1)\big\} + x_{cl}^T(k)Qx_{cl}(k) \\
&\qquad - x_{cl}^T(k)Px_{cl}(k) - x_{cl}^T(k-1)Qx_{cl}(k-1) \\
&\quad = (A_{cl0}x_{cl}(k) + A_{dcl0}x_{cl}(k-1))^T P(A_{cl0}x_{cl}(k) + A_{dcl0}x_{cl}(k-1)) \\
&\qquad + \mathbb{E}\big\{(\alpha(k)-\bar{\alpha})^2\big\}(A_{cl1}x_{cl}(k) + A_{dcl1}x_{cl}(k-1))^T P(A_{cl1}x_{cl}(k) \\
&\qquad + A_{dcl1}x_{cl}(k-1)) \\
&\qquad + \mathbb{E}\big\{(\beta(k)-\bar{\beta})^2\big\}(A_{cl2}x_{cl}(k) + A_{dcl2}x_{cl}(k-1))^T P(A_{cl2}x_{cl}(k) \\
&\qquad + A_{dcl2}x_{cl}(k-1)) \\
&\qquad + x_{cl}^T(k)Qx_{cl}(k) - x_{cl}^T(k)Px_{cl}(k) - x_{cl}^T(k-1)Qx_{cl}(k-1).
\end{aligned} \tag{22}$$

Noting that $\mathbb{E}\{(\alpha(k)-\bar{\alpha})^2\} = (1-\bar{\alpha})\bar{\alpha} := a$ and $\mathbb{E}\{(\beta(k)-\bar{\beta})^2\} = (1-\bar{\beta})\bar{\beta} := b$, we have

$$\begin{aligned}
&E\{V(\eta(k+1))\,|\,V(\eta(k))\} - V(\eta(k)) \\
&\quad = (A_{cl0}x_{cl}(k) + A_{dcl0}x_{cl}(k-1))^T P(A_{cl0}x_{cl}(k) + A_{dcl0}x_{cl}(k-1)) \\
&\qquad + a(A_{cl1}x_{cl}(k) + A_{dcl1}x_{cl}(k-1))^T P(A_{cl1}x_{cl}(k) + A_{dcl1}x_{cl}(k-1)) \\
&\qquad + b(A_{cl2}x_{cl}(k) + A_{dcl2}x_{cl}(k-1))^T P(A_{cl2}x_{cl}(k) + A_{dcl2}x_{cl}(k-1)) \\
&\qquad + x_{cl}^T(k)Qx_{cl}(k) - x_{cl}^T(k)Px_{cl}(k) - x_{cl}^T(k-1)Qx_{cl}(k-1) \\
&\quad = \eta^T(k)\Lambda\eta(k),
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\Lambda &= \begin{bmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2^T & \Lambda_3 \end{bmatrix}, \\
\Lambda_1 &= A_{cl0}^T P A_{cl0} + aA_{cl1}^T P A_{cl1} + bA_{cl2}^T P A_{cl2} + Q - P, \\
\Lambda_2 &= A_{cl0}^T P A_{dcl0} + aA_{cl1}^T P A_{dcl1} + bA_{cl2}^T P A_{dcl2}, \\
\Lambda_3 &= A_{dcl0}^T P A_{dcl0} + aA_{dcl1}^T P A_{dcl1} + bA_{dcl2}^T P A_{dcl2} - Q.
\end{aligned}$$

By Schur complement, equation (20) is equivalent to $\Lambda < 0$, we know from equation (23) that

$$\begin{aligned}
&\mathbb{E}\{V(\eta(k+1))\,|\,V(\eta(k))\} - V(\eta(k)) \\
&\quad = \eta^T(k)\Lambda\eta(k) \le -\lambda_{\min}(-\Lambda)\eta^T(k)\eta(k) < -\varepsilon\eta^T(k)\eta(k),
\end{aligned} \tag{24}$$

where

$$0 < \epsilon < \min\{\lambda_{\min}(-\Lambda), \sigma\}, \quad \sigma := \max\{\lambda_{\max}(P), \lambda_{\max}(Q)\}. \tag{25}$$

From (24), we have

$$\begin{aligned}&\mathbb{E}\{V(\eta(k+1)) \,|\, V(\eta(k))\} - V(\eta(k)) < -\epsilon\eta^T(k)\eta(k) < -\frac{\epsilon}{\sigma}V(\eta(k))\\&:= -\psi V(\eta(k)).\end{aligned} \tag{26}$$

Therefore, by Definition 1, it can be verified from Lemma 1 that the closed-loop system (14) is exponentially mean-square stable. This completes the proof. □

The following theorem provides a sufficient condition for the closed-loop system (14) to be asymptotically mean-square stable and for the controlled output $z(k)$ to satisfy the H_∞ disturbance attenuation in equation (16).

Theorem 2: *Given a scalar $\gamma > 0$ and the controller parameters A_K, B_K, and C_K. The system* (14) *is exponentially mean-square stable and the H_∞-norm constraint* (16) *is achieved for all nonzero $w(k)$, if there exist positive definite matrices P and Q satisfying*

$$\begin{pmatrix} Q-P & * & * & * & * & * & * & * \\ 0 & -Q & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ PA_{cl0} & PA_{dcl0} & PB_{cl} & -P & * & * & * & * \\ aPA_{cl1} & aPA_{dcl1} & 0 & 0 & -aP & * & * & * \\ bPA_{cl2} & bPA_{dcl2} & 0 & 0 & 0 & -bP & * & * \\ C_{cl0} & C_{dcl0} & D_{cl} & 0 & 0 & 0 & -I & * \\ bC_{cl2} & bC_{dcl2} & 0 & 0 & 0 & 0 & 0 & -bI \end{pmatrix} < 0. \tag{27}$$

Proof: It is obvious that equation (27) implies equation (20), hence it follows from Theorem 1 that the system (14) is exponentially mean-square stable. Next, for any nonzero $w(k)$, it follows from equations (14) and (23) that Bouhtouri et al. (1999)

$$\begin{aligned}&\mathbb{E}\{V(\eta(k+1))\} - \mathbb{E}\{V(\eta(k))\} + \mathbb{E}\{z^T(k)z(k)\} - \gamma^2\mathbb{E}\{w^T(k)w(k)\}\\&= \xi^T(k)\Xi\xi(k),\end{aligned} \tag{28}$$

where

$$\xi(k) = \begin{bmatrix} x_{cl}(k) \\ x_{cl}(k-1) \\ w(k) \end{bmatrix}, \quad \Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix},$$

$$\begin{aligned}\Xi_{11} &= A_{cl0}^T PA_{cl0} + aA_{cl1}^T PA_{cl1} + bA_{cl2}^T PA_{cl2} + C_{cl0}^T C_{cl0} + bC_{cl2}^T C_{cl2} + Q - P,\\ \Xi_{12} &= A_{cl0}^T PA_{dcl0} + aA_{cl1}^T PA_{dcl1} + bA_{cl2}^T PA_{dcl2} + C_{cl0}^T C_{dcl0} + bC_{cl2}^T C_{dcl2},\\ \Xi_{13} &= A_{cl0}^T PB_{cl} + C_{cl0}^T D_{cl},\end{aligned}$$

$$\begin{aligned}
\Xi_{22} &= A_{dcl0}^T P A_{dcl0} + a A_{dcl1}^T P A_{dcl1} + b A_{dcl2}^T P A_{dcl2} + C_{dcl0}^T C_{dcl0} \\
&\quad + b C_{dcl2}^T C_{dcl2} - Q, \\
\Xi_{23} &= A_{dcl0}^T P B_{cl} + C_{dcl0}^T D_{cl}, \\
\Xi_{33} &= B_{cl}^T P B_{cl} + D_{cl}^T D_{cl} - \gamma^2 I.
\end{aligned}$$

By Schur complement, equation (27) is equivalent to $\Xi < 0$. Thus, we have

$$\mathbb{E}\{V(\eta(k+1))\} - \mathbb{E}\{V(\eta(k))\} + \mathbb{E}\{z^T(k)z(k)\} - \gamma^2 \mathbb{E}\{w^T(k)w(k)\} < 0. \quad (29)$$

Now, summing up equation (29) from 0 to ∞ with respect to k yields

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|z(k)\|^2\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|w(k)\|^2\} + \mathbb{E}\{V(0)\} - \mathbb{E}\{V(\infty)\}. \quad (30)$$

Since $\eta(0) = 0$ and the system (14) is exponentially mean-square stable, it is straightforward to see that

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|z(k)\|^2\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|w(k)\|^2\}. \quad (31)$$

This ends the proof. □

Next, the controller design problem is solved in the following theorem and the controller parameters are given in terms of the solution to a matrix ineqaulity.

Theorem 3: *Given a scalar $\gamma > 0$. The system (14) is asymptotically mean-square stable and the H_∞-norm constraint (16) is achieved for all nonzero $w(k)$, if there exist positive definite matrices $S = S^T > 0$, $V = V^T > 0$, $\widehat{Q}_1 = \widehat{Q}_1^T > 0$ and $\widehat{Q}_3 = \widehat{Q}_3^T > 0$, real matrices $\widehat{Q}_2$, $\overline{A}_c$, $\overline{B}_c$ and $\overline{C}_c$ such that*

$$\begin{bmatrix} \Sigma_1 & * \\ \Sigma_2 & \Sigma_3 \end{bmatrix} < 0, \quad (32)$$

where

$$\Sigma_1 = \begin{bmatrix} \widehat{Q}_1 - S^{-1} & * & * & * & * \\ \widehat{Q}_2 - S^{-1} & \widehat{Q}_3 - V^{-1} & * & * & * \\ 0 & 0 & -\widehat{Q}_1 & * & * \\ 0 & 0 & -\widehat{Q}_2 & -\widehat{Q}_3 & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \quad (33)$$

$$\Sigma_2 = \begin{bmatrix} A + \bar{\beta} B_2 \overline{C}_c & A & (1-\bar{\beta}) B_2 \overline{C}_c & 0 & B_1 \\ \Omega_1 & A + \bar{\alpha} \overline{B}_c C_2 & \Omega_2 & (1-\bar{\alpha}) \overline{B}_c C_2 & B_1 \\ 0 & 0 & 0 & 0 & 0 \\ a \overline{B}_c C_2 & a \overline{B}_c C_2 & -a \overline{B}_c C_2 & -a \overline{B}_c C_2 & 0 \\ b B_2 \overline{C}_c & 0 & -b B_2 \overline{C}_c & 0 & 0 \\ b B_2 \overline{C}_c & 0 & -b B_2 \overline{C}_c & 0 & 0 \\ C_1 + \bar{\beta} D_{12} \overline{C}_c & C_1 & (1-\bar{\beta}) D_{12} \overline{C}_c & 0 & 0 \\ b D_{12} \overline{C}_c & 0 & -b D_{12} \overline{C}_c & 0 & 0 \end{bmatrix} < 0, \quad (34)$$

$$\Sigma_3 = \begin{bmatrix} -S & * & * & * & * & * & * & * \\ -V & -V & * & * & * & * & * & * \\ 0 & 0 & -aS & * & * & * & * & * \\ 0 & 0 & -aV & -aV & * & * & * & * \\ 0 & 0 & 0 & 0 & -bS & * & * & * \\ 0 & 0 & 0 & 0 & -bV & -bV & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -bI \end{bmatrix} < 0, \tag{35}$$

with

$$\Omega_1 = A + \bar{\alpha}\overline{B}_c C_2 + \bar{\beta} B_2 \overline{C}_c + \overline{A}_c,$$
$$\Omega_2 = (1-\bar{\alpha})\overline{B}_c C_2 + (1-\bar{\beta}) B_2 \overline{C}_c,$$

Moreover, the controller parameters are given by

$$A_K = X_{12}^{-1} V^{-1} \overline{A}_c (VS^{-1} - I)^{-1} V X_{12}, \tag{36}$$
$$B_K = X_{12}^{-1} V^{-1} \overline{B}_c, \tag{37}$$
$$C_K = \overline{C}_c (VS^{-1} - I)^{-1} V X_{12} \tag{38}$$

where the matrix X_{12} comes from the factorisation $I - V^{-1}S = X_{12}Y_{12}^T < 0$.

Proof: Recall that our goal is to derive the expression of the controller parameters from (8). To do this, we partition P and P^{-1} as

$$P = \begin{bmatrix} R & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} S & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}, \tag{39}$$

where the partitioning of P and P^{-1} is compatible with that of A_{cl0} defined in equation (10). Define

$$T_1 = \begin{bmatrix} S & I \\ Y_{12}^T & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} I & R \\ 0 & X_{12}^T \end{bmatrix}, \tag{40}$$

which imply that $PT_1 = T_2$ and $T_1^T P T_1 = T_1^T T_2$.

By applying the congruence transformations $\mathrm{diag}\{T_1, T_1, I, T_1, T_1, T_1, I, I\}$ to equation (27) and defining

$$T_1^T Q T_1 = \begin{bmatrix} S\widehat{Q}_1 S & * \\ \widehat{Q}_2 S & \widehat{Q}_3 \end{bmatrix},$$

we obtain

$$\begin{bmatrix} \Sigma_{11} & * \\ \Sigma_{12} & \Sigma_{13} \end{bmatrix} < 0, \tag{41}$$

where

$$\Sigma_{11} = \begin{bmatrix} S\widehat{Q}_1S - S & * & * & * & * \\ \widehat{Q}_2S - I & \widehat{Q}_3 - R & * & * & * \\ 0 & 0 & -S\widehat{Q}_1S & * & * \\ 0 & 0 & -\widehat{Q}_2S & -\widehat{Q}_3 & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{42}$$

$$\Sigma_{12} = \begin{bmatrix} AS + \bar{\beta}B_2C_KY_{12}^T & A & (1-\bar{\beta})B_2C_KY_{12}^T & 0 & B_1 \\ \Omega_{11} & RA + \bar{\alpha}X_{12}B_KC_2 & \Omega_{12} & (1-\bar{\alpha})X_{12}B_KC_2 & RB_1 \\ 0 & 0 & 0 & 0 & 0 \\ aX_{12}B_KC_2S & aX_{12}B_KC_2 & -aX_{12}B_KC_2S & -aX_{12}B_KC_2 & 0 \\ bB_2C_KY_{12}^T & 0 & -bB_2C_KY_{12}^T & 0 & 0 \\ bRB_2C_KY_{12}^T & 0 & -bRB_2C_KY_{12}^T & 0 & 0 \\ C_1S + \bar{\beta}D_{12}C_KY_{12}^T & C_1 & (1-\bar{\beta})D_{12}C_KY_{12}^T & 0 & 0 \\ bD_{12}C_KY_{12}^T & 0 & -bD_{12}C_KY_{12}^T & 0 & 0 \end{bmatrix} < 0, \tag{43}$$

$$\Sigma_{13} = \begin{bmatrix} -S & * & * & * & * & * & * & * \\ -I & -R & * & * & * & * & * & * \\ 0 & 0 & -aS & * & * & * & * & * \\ 0 & 0 & -aI & -aR & * & * & * & * \\ 0 & 0 & 0 & 0 & -bS & * & * & * \\ 0 & 0 & 0 & 0 & -bI & -bR & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -bI \end{bmatrix} < 0, \tag{44}$$

with

$$\Omega_{11} = RAS + \bar{\alpha}X_{12}B_KC_2S + \bar{\beta}RB_2C_KY_{12}^T + X_{12}A_KY_{12}^T,$$
$$\Omega_{12} = (1-\bar{\alpha})X_{12}B_KC_2S + (1-\bar{\beta})RB_2C_KY_{12}^T.$$

By applying the congruence transformations $\mathrm{diag}\{S^{-1}, I, S^{-1}, I, I, I, R^{-1}, I, R^{-1}, I, R^{-1}, I, I\}$ to equation (41), we obtain

$$\begin{bmatrix} \Sigma_{21} & * \\ \Sigma_{22} & \Sigma_{23} \end{bmatrix} < 0, \tag{45}$$

where

$$\Sigma_{21} = \begin{bmatrix} \widehat{Q}_1 - S^{-1} & * & * & * & * \\ \widehat{Q}_2 - S^{-1} & \widehat{Q}_3 - R & * & * & * \\ 0 & 0 & -\widehat{Q}_1 & * & * \\ 0 & 0 & -\widehat{Q}_2 & -\widehat{Q}_3 & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{46}$$

$$\Sigma_{22} = \left[\begin{matrix} A + \bar{\beta} B_2 C_K Y_{12}^T S^{-1} & A \\ \Omega_{21} & A + \bar{\alpha} R^{-1} X_{12} B_K C_2 \\ 0 & 0 \\ aR^{-1} X_{12} B_K C_2 & aR^{-1} X_{12} B_K C_2 \\ bB_2 C_K Y_{12}^T S^{-1} & 0 \\ bB_2 C_K Y_{12}^T S^{-1} & 0 \\ C_1 + \bar{\beta} D_{12} C_K Y_{12}^T S^{-1} & C_1 \\ bD_{12} C_K Y_{12}^T S^{-1} & 0 \end{matrix}\right.$$

$$\times \left.\begin{matrix} (1-\bar{\beta}) B_2 C_K Y_{12}^T S^{-1} & 0 & B_1 \\ \Omega_{22} & (1-\bar{\alpha}) R^{-1} X_{12} B_K C_2 & B_1 \\ 0 & 0 & 0 \\ -aR^{-1} X_{12} B_K C_2 & -aR^{-1} X_{12} B_K C_2 & 0 \\ -bB_2 C_K Y_{12}^T S^{-1} & 0 & 0 \\ -bB_2 C_K Y_{12}^T S^{-1} & 0 & 0 \\ (1-\bar{\beta}) D_{12} C_K Y_{12}^T S^{-1} & 0 & 0 \\ -bD_{12} C_K Y_{12}^T S^{-1} & 0 & 0 \end{matrix}\right] < 0, \tag{47}$$

$$\Sigma_{23} = \begin{bmatrix} -S & * & * & * & * & * & * & * \\ -R^{-1} & -R^{-1} & * & * & * & * & * & * \\ 0 & 0 & -aS & * & * & * & * & * \\ 0 & 0 & -aR^{-1} & -aR^{-1} & * & * & * & * \\ 0 & 0 & 0 & 0 & -bS & * & * & * \\ 0 & 0 & 0 & 0 & -bR^{-1} & -bR^{-1} & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -bI \end{bmatrix} < 0, \tag{48}$$

with

$$\Omega_{21} = A + \bar{\alpha} R^{-1} X_{12} B_K C_2 + \bar{\beta} B_2 C_K Y_{12}^T S^{-1} + R^{-1} X_{12} A_K Y_{12}^T S^{-1},$$
$$\Omega_{22} = (1-\bar{\alpha}) R^{-1} X_{12} B_K C_2 + (1-\bar{\beta}) B_2 C_K Y_{12}^T S^{-1}.$$

Now define the change of controller parameters as follows:

$$\overline{A}_c = R^{-1} X_{12} A_K Y_{12}^T S^{-1}, \quad \overline{B}_c = R^{-1} X_{12} B_K, \quad \overline{C}_c = C_K Y_{12}^T S^{-1} \tag{49}$$

and let

$$V = R^{-1}, \tag{50}$$

we can get (32) immediately.

Furthermore, if the matrix inequality (32) is feasible, then we have $\left[\begin{smallmatrix} -S^{-1} & -S^{-1} \\ -S^{-1} & -V^{-1} \end{smallmatrix}\right] < 0$, i.e., $\left[\begin{smallmatrix} S & I \\ I & R \end{smallmatrix}\right] > 0$. It follows directly from $PP^{-1} = I$ that $I - RS = X_{12}Y_{12}^T < 0$. Hence, one can always find square and nonsingular X_{12} and Y_{12} (Scherer et al., 1997). Therefore, equations (36)–(38) are obtained from equation (49), which concludes the proof. □

Remark 1: In view of equations (36)–(38), we could make the linear transformation on the controller

$$\bar{x}(k) = VX_{12}\hat{x}(k), \tag{51}$$

and then obtain a new representation form of the filter as follows:

$$\begin{cases} \bar{x}(k+1) = \overline{A}_K\bar{x}(k) + \overline{B}_K y_c(k), \\ u(k) = \overline{C}_K\bar{x}(k), \end{cases} \tag{52}$$

where

$$\overline{A}_K = \overline{A}_c(VS^{-1} - I)^{-1}, \quad \overline{B}_K = \overline{B}_c, \quad \overline{C}_K = \overline{C}_c(VS^{-1} - I)^{-1}. \tag{53}$$

We can now see from (53) that, the controller parameters can be obtained directly by solving equation (32) without solving $I - RS = X_{12}Y_{12}^T$ for X_{12}.

However, (32) is a nonlinear matrix inequality. Therefore, we can not solve it by standard LMI ToolBox (Boyd et al., 1994). Now we denote

$$W = S^{-1}, \tag{54}$$

then (32) can be changed as

$$\begin{bmatrix} \Sigma_4 & * \\ \Sigma_2 & \Sigma_3 \end{bmatrix} < 0, \tag{55}$$

with

$$WS = I, \quad VR = I, \tag{56}$$

where

$$\Sigma_4 = \begin{bmatrix} \widehat{Q}_1 - W & * & * & * & * \\ \widehat{Q}_2 - W & \widehat{Q}_3 - R & * & * & * \\ 0 & 0 & -\widehat{Q}_1 & * & * \\ 0 & 0 & -\widehat{Q}_2 & -\widehat{Q}_3 & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{57}$$

and Σ_2 and Σ_3 see equations (34) and (35), respectively.

Thus we can see that the nonlinear matrix inequality (32) is equivalent to a linear matrix inequality (55) with the equality constraints (56). This problem can be solved via the Cone Complementarity Linearisation Method (CCLM) (El Ghaoui et al., 1997) or the Sequential Linear Programming Matrix Method (SLPMM) (Leibfritz, 2001). The principle of CCLM and SLPMM algorithm is that if the LMI $\begin{bmatrix} W & I \\ I & S \end{bmatrix} \geq 0$ is feasible in the $n \times n$ matrix variables $W > 0$ and $S > 0$, then trace$(WS) \geq n$ and trace$(WS) = n$ if and only if $WS = I$. So the equality constraints in equation (56) can be weakened to the following semi-definite programming relaxations:

$$\begin{bmatrix} W & I \\ I & S \end{bmatrix} \geq 0, \quad \begin{bmatrix} V & I \\ I & R \end{bmatrix} \geq 0. \tag{58}$$

We now extend the SLPMM to solve the above problem by the iterative LMI approach (Lin et al., 2004). The algorithm can be summarised as follows:

Step 1: Given the maximum iteration times N and the iteration accuracy ϵ.

Step 2: Find a feasible solution to equations (55) and (58), and let $W^{(k)} = W$, $S^{(k)} = S$, $V^{(k)} = V$, $R^{(k)} = R$, $k = 0$.

Step 3: Find a set of optimal solution $\widehat{Q}_1, \widehat{Q}_2, \widehat{Q}_3, \overline{A}_c, \overline{B}_c, \overline{C}_c, W, S, V, R$ such that

$$\min_{\text{subject to (55) and (58)}} \text{trace}(WS^{(k)} + W^{(k)}S + VR^{(k)} + V^{(k)}R) + \delta, \quad \delta = \gamma^2.$$

Step 4: Substitute the obtained matrix variables $(\widehat{Q}_1, \widehat{Q}_2, \widehat{Q}_3, \overline{A}_c, \overline{B}_c, \overline{C}_c, W, S, R, V, \gamma)$ into (32). If condition (32) is satisfied, with

$$|\text{trace}(WS^{(k)} + W^{(k)}S + VR^{(k)} + V^{(k)}R) - 4n| \leq \epsilon,$$

then output the feasible solutions $(\widehat{Q}_1, \widehat{Q}_2, \widehat{Q}_3, \overline{A}_c, \overline{B}_c, \overline{C}_c, W, S, R, V, \gamma)$. EXIT.

Step 5: If $k > N$, EXIT.

Step 6: Compute $\rho^* \in [0, 1]$ such that

$$\begin{aligned} \min_{\rho \in [0,1]} \big\{ &\text{trace}\big(W^{(k)} + \rho\big(W - W^{(k)}\big)\big)\big(S^{(k)} + \rho\big(S - S^{(k)}\big)\big) \\ &+ \text{trace}\big(V^{(k)} + \rho\big(V - V^{(k)}\big)\big)\big(R^{(k)} + \rho\big(R - R^{(k)}\big)\big)\big\}. \end{aligned}$$

Step 7: Let

$$\begin{aligned} W^{(k)} &= W^{(k)} + \rho^*\big(W - W^{(k)}\big), \quad S^{(k)} = S^{(k)} + \rho^*\big(S - S^{(k)}\big), \\ V^{(k)} &= V^{(k)} + \rho^*\big(V - V^{(k)}\big), \quad R^{(k)} = R^{(k)} + \rho^*\big(R - R^{(k)}\big), \end{aligned}$$

and $k = k + 1$, go to Step 3.

4 An illustrative example

In this section, we aim to demonstrate the effectiveness and applicability of the proposed method. For this purpose, we consider the system described by (1) with parameters as follows:

$$A = \begin{bmatrix} -0.21 & -0.02 & 0.01 \\ 2.4 & 0.81 & 0.059 \\ -79 & 2.6 & 0.01 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1 & -1 \\ -10 & 1 \\ -7 & -5 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}.$$

We assumed the random communication delays probabilities $\bar{\alpha} = 0.9$ and $\bar{\beta} = 0.9$. Solving the optimisation problem using SPLMM with SeDuMi solver, yields the minimum value $\gamma_{\min} = 0.2606$ and the corresponding controller parameters are

$$A_K = \begin{bmatrix} 0.0225 & -0.0681 & -0.0009 \\ 2.1243 & -0.6375 & 0.0011 \\ -85.3292 & 4.0722 & -0.0193 \end{bmatrix}, \quad B_K = \begin{bmatrix} 0.0494 & 0.0106 \\ -1.1151 & -0.3955 \\ 34.2128 & 0.7867 \end{bmatrix},$$

$$C_K = \begin{bmatrix} -0.0637 & -0.1118 & -0.0069 \\ 0.5342 & 0.1971 & 0.0001 \end{bmatrix}.$$

If the random communication delays probabilities are changed to $\bar{\alpha} = 0.8$ and $\bar{\beta} = 0.8$, then the optimal disturbance attenuation is $\gamma_{\min} = 0.3229$, and the corresponding controller parameters are

$$A_K = \begin{bmatrix} -0.0670 & -0.0583 & 0.0025 \\ 2.7356 & 0.2111 & 0.0283 \\ -86.3615 & 2.0655 & -0.0270 \end{bmatrix}, \quad B_K = \begin{bmatrix} 0.0187 & -0.0004 \\ -0.6228 & -0.1889 \\ 18.0708 & -1.2288 \end{bmatrix},$$

$$C_K = \begin{bmatrix} 0.0086 & -0.0651 & -0.0047 \\ 0.1587 & 0.1011 & -0.0014 \end{bmatrix}.$$

If the random communication delays probabilities are changed to $\bar{\alpha} < 0.7900$ and $\bar{\beta} < 0.7789$, then the feasible solutions cannot be found.

From the above simulation results, we can see that the H_∞ performance with the probability of data communication delays is bigger than that without data communication delays. We can conclude that the H_∞ disturbance attenuation performance will deteriorate if the data packet communication delays are severe.

5 Conclusions

In this paper, an iterative LMI approach has been presented to investigate the control problem for NCSs with random communication delays. A dynamic output controller has been designed to achieve a desired H_∞ disturbance attenuation level based on a stochastic parameter system that the resulting closed-loop NCS is modelled as. The optimal H_∞ disturbance attenuation and the controller parameters are obtained by solving the iterative LMI problem. Simulation results have demonstrated the feasibility of our control scheme. Our future research will extend the recent results to NCSs with multiple random delays.

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